MATHEMATICS

Duration: Three Hours *Maximum Marks: 150*

➢ **Read the following instructions carefully.**

1. This question paper contains all objective questions. Q. 1 to Q. 30 carries one mark each and Q. 31 to Q.80 carries two marks each. Q. 81 to Q. 85 each contains part "a" and "b". In these questions, part "a" as well as "b" carry two marks each.
2. Answer all the questions.
3. Questions must be answered on special machine gradable Objective Response Sheet (ORS) by darkening the appropriate bubble (marked A, B, C, D) against the question number on the left hand side of the ORS, using HB pencil. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be negative marking. In Q.1 to Q.30, 0.25 mark will be deducted for each wrong answer and in Q. 31 to Q. 80, 0.5 mark will be deduced for each wrong answer. In Q.81 to Q.85, for the part "a", 0.5 mark will be deducted for a wrong answer. Marks for correct answers to part "b" of Q. 81 to Q.85 will be given only if the answer to the corresponding part "a" is correct. However, there is no negative marking for part "b" of Q. 81 to Q. 85. More than one answer bubbled against a question will be deemed as an incorrect response.
5. Write your registration number, name and name of the center at the specified locations on the right half of the ORS.
6. Using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your paper code.
7. Calculator is allowed in the examination hall.
8. Charts, graph sheets or tables are not allowed.
9. Use the blank pages given at the end of the question paper for rough work.
10. This question paper contains 28 printed pages including pages for rough work. Please check all pages and report, if there is any discrepancy.

The symbols N, Z, R and C denote the set of natural numbers, integers, real numbers and complex numbers, respectively throughout the paper.

ONE MARKS QUESTIONS (1-30)

1. The set of all $x \in R$ for which the vectors $(1, x, 0)$, $(0, x^2, 1)$ and $(0, 1, x)$ are linearly independent in R^3 is
   a. $\{x \in R : x = 0\}$
   b. $\{x \in R : x \neq 0\}$
   c. $\{x \in R : x \neq 1\}$
   d. $\{x \in R : x \neq -1\}$

2. Consider the vector space R^3 and the maps $f, g : R^3 \to R^3$ defined by $f(x, y, z) = (x, |y|, z)$ and $f(x, y, z) = (x+1, y-1, z)$. Then
   a. Both f and g are linear
   b. Neither f nor g is linear
   c. g is linear but not f
   d. f is linear but not g

3. Let $M = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{pmatrix}$. Then
   a. M is diagonalizable but not M^2
   b. M^2 is diagonalizable but not M
   c. Both M and M^2 are diagonalizable
   d. Neither M nor M^2 is diagonalizable

4. Let M be a skew symmetric, orthogonal real matrix, The only possible eigen values are
   a. –1, 1
   b. – i, i
   c. 0
   d. 1, i

5. The principal value of $\log\left(i^{\frac{1}{4}}\right)$ is
   a. $i\pi$
   b. $\frac{i\pi}{2}$
   c. $\frac{i\pi}{4}$
   d. $\frac{i\pi}{8}$

6. Consider the functions $f(z) = x^2 + iy^2$ and $g(z) = x^2 + y^2 + ixy$. At z = 0.
   a. f is analytic but not g

b. g is analytic but not f
c. Both f and g are analytic
d. Neither f nor g is analytic

7. The coefficient of $\frac{1}{z}$ in the expansion of $\log\left(\frac{z}{z-1}\right)$, valid in $|z|>1$ is
a. -1
b. 1
c. $-\frac{1}{2}$
d. $\frac{1}{2}$

8. Under the usual topology in R^3, if $O=\{(x,y,z)\in R^3 : x^2+y^2<1\}$ and $F=\{(x,y,z)\in R^3 : z=0\}$, then $O\cap F$ is
a. Both open and closed
b. Neither open nor closed
c. Open but nor closed
d. Closed but not open

9. Suppose E is a non measurable subset of $[\theta,1]$. Let P = $E^\circ\cup\left\{\frac{1}{n}:n\in N\right\}$ and $Q=\bar{E}\cup\left\{\frac{1}{n}:n\in N\right\}$ where E° is the interior of E and $\bar{E}$ is the closure of E. Then
a. P is measurable but not Q
b. Q is measurable but not P
c. Both P and Q are measurable
d. Neither P nor Q is measurable

10. The value of $\int_0^\pi\int_x^\pi\int_0^2 \frac{\sin y}{y}\,dz\,dy\,dx$ is
a. -2
b. 2
c. -4
d. 4

11. Let $S=\left\{\frac{1}{n}:n\in N\right\}\cup\{0\}$ and $T=\left\{n+\frac{1}{n}:n\in N\right\}$ be the subsets of the metric space R with the usual metric. Then
a. S is complete but not T
b. T is complete but not S
c. Both T and S are complete
d. Neither T nor S is complete

12. In a sufficiently small neighborhood around x = 2, the differential equation $\frac{dy}{dx}=\frac{y}{\sqrt{x}}, y(2)=4$ has
a. No solution
b. A unique solution
c. Exactly two solutions
d. Infinitely many solutions

13. The set of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}-\frac{d^2y}{dx^2}=0$ is
a. $\{1,x,e^x,e^{-x}\}$
b. $\{1,x,e^{-x},xe^{-x}\}$
c. $\{1,x,e^x,xe^x\}$
d. $\{1,x,e^x,xe^{-x}\}$

14. For the differential equation $x^2(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$
a. X = 1 is an ordinary point
b. X = 1 is a regular singular point
c. X = 0 is an irregular singular point
d. X = 0 is an ordinary point

15. Let D_8 denote the group of symmetries of square (dihedral group). The minimal number of generators for D_8 is
a. 1
b. 2
c. 4
d. 8

16. Let the set $\frac{Z}{nZ}$ denote the ring of integers modulo n number addition and multiplication modulo n. Then $\frac{Z}{9Z}$ is not a sub ring of $\frac{Z}{12Z}$ because
a. $Z/9Z$ is not a subset of $Z/12Z$
b. G.C.D. (9, 12) = 3 ≠1
c. 12 is not a power of 3
d. 9 does not divide 12

17. Let $C[0,1]$, be the space of all continuous real valued functions on $[0,1]$. The identity map $I:(C[0,1],\|\cdot\|_\infty)\to(C[0,1],\|\cdot\|_1)$ is
a. Continuous but not open

b. Open but not continuous
c. Both continuous and open
d. Neither continuous nor open

18. Consider the Hilbert space $l^2 = \{(x_1, x_2.....), x_1 \in C$ for all i and $\sum_{i=1}^{\infty} |x_i|^2 < \infty\}$ with the inner product $\langle (x_1, x_2.....)(y_1, y_2.....)\rangle = \sum_{i=1}^{\infty} x_i \bar{y}_i$. Define $T : l^2 \to l^2$ by $T((x_1, x_2,....)) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, ...\right)$. Then T is
a. Neither self-ad joint nor unitary
b. Both Self-ad joint and unitary
c. Unitary but not –ad joint
d. Self-ad joint but unitary

19. An iterative method of find the nth root $(n \in N)$ of a positive number a is given by $x_{k+1} = \frac{1}{2}\left[x_k + \frac{a}{x_k^{n-1}}\right]$. A value of n for which this iterative method fails to converge is
a. 1
b. 2
c. 3
d. 8

20. Suppose the function $u(x)$ interpolates $f(x)$ at $x_0, x_1, x_2, ..., x_{n-1}$ and the function $v(x)$ interpolates $f(x)$ at $x_1, x_2, ...x_{n-1}$. Then , a function $F(x)$ which interpolates $f(x)$ at all the points $x_0, x_1, x_2, ..., x_{n-1}, x_n$ is given by
a. $E(x) = \frac{(x_n - x)u(x) - (x - x_0)v(x)}{(x_n - x_0)}$
b. $F(x) = \frac{(x_n - x)u(x) + (x - x_0)v(x)}{(x_n - x_0)}$
c. $F(x) = \frac{(x_n - x)v(x) + (x - x_0)u(x)}{(x_n - x_0)}$
d. $F(x) = \frac{(x_n - x)v(x) - (x - x_0)u(x)}{(x_n - x_0)}$

21. The integral surface of the partial differential equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ satisfying the condition $u(1, y) = y$ is given by
a. $u(x, y) = \frac{y}{x}$
b. $u(x, y) = \frac{2y}{x+1}$
c. $u(x, y) = \frac{y}{2-x}$
d. $u(x, y) = y + x - 1$

22. If $f(x)$ and $g(y)$ are arbitrary functions, then the general solution of the partial differential equation $u\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} = 0$ is given by
a. $u(x, y) = f(x) + g(y)$
b. $u(x, y) = f(x + y) + g(x - y)$
c. $u(x, y) = f(x)g(y)$
d. $u(x, y) = xg(y) + yf(x)$

23. A bead slides on a smooth rood which is rotating about one end is a vertical plane with uniform angular velocity ω. If g denotes the acceleration due to gravity, then the Lagrange equation of motion is
a. $\ddot{r} = r\omega^2 - g \sin \omega t$
b. $\ddot{r} = r\omega^2 - g \cos \omega t$
c. $\ddot{r} = -g \sin \omega t$
d. $\ddot{r} = -g \cos \omega t$

24. The Lagrangian L of a dynamical system with two degree of freedom is given by $L = \alpha + \beta q_1 + \gamma q_2$ where α, β and γ are functions of the generalized coordinates q_1, q_2 only. If p_1, p_2 denote the generalized momenta, then Hamiltonian H
a. Depends on p_1, p_2 but not on p_1, p_2
b. Depends on q_1, q_2 but not on p_1, p_2
c. Depends on p_1, q_1 but not on p_2, q_2
d. Is independent of p_1, p_2, q_1, q_2

25. Let $A_1, A_2, ...A_n$ be n independent events which the probability of occurrence of the event A_i given by $P(A_i) = 1 - \frac{1}{\alpha^i}, \alpha > l,\ i = 1, 2, ...n$. Then the probability that at least one of the events occurs is

a. $1-\dfrac{1}{\alpha^{\frac{n(n+1)}{2}}}$

b. $\dfrac{1}{\alpha^{\frac{n(n+1)}{2}}}$

c. $\dfrac{1}{\alpha^{n}}$

d. $1-\dfrac{1}{\alpha^{n}}$

26. The life time of two brands of bulbs X and Y are exponentially distributed with a mean life time of 100 hours. Bulb X is switched on 15 hours after bulb Y has been switched on. The probability that the bulb X fails before Y is

a. $\dfrac{15}{100}$

b. $\dfrac{1}{2}$

c. $\dfrac{85}{100}$

d. 0

27. A random sample of size n is chose from a population with probability density function $f(x,\theta)=\begin{cases}\dfrac{1}{2}e^{-(x-\theta)}, & x\geq\theta\\ \dfrac{1}{2}e^{(x-\theta)}, & x<\theta\end{cases}$

Then, the maximum likelihood estimator of θ is the

a. Mean of the sample
b. Standard deviation of the sample
c. Median of the sample
d. Maximum of the sample

28. Consider the following linear Programming Problem (LPP):

Minimize $z=2x_1+3x_2+x_3$

Subject to $x_1+2x_2+2x_3-x_4+x_5=3$

$2x_1+3x_2+4x_3+x_6=6$

$x_i\geq 0,\quad i=1,2,......6.$

A non degenerate basic feasible solution $(x_1,x_2,x_3,x_4,x_5,x_6)$ is

a. $(1,0,1,0,0,0)$

b. $(1,0,0,0,0,7)$

c. $(0,0,0,0,3,6)$

d. $(3,0,0,0,0,0)$

29. The unit cost c_{ij} of producing product i at plant j is given by the matrix:

$$\begin{pmatrix}14 & 12 & 16\\ 21 & 9 & 17\\ 9 & 7 & 5\end{pmatrix}$$

The total cost of optimal assignment is

a. 20
b. 22
c. 25
d. 28

30. The eigen values λ of the integral equation $y(x)=\lambda\int_0^{2\pi}\sin(x+t)(t)\,dt$ are

a. $\dfrac{1}{2\pi},-\dfrac{1}{2\pi}$

b. $\dfrac{1}{\pi},-\dfrac{1}{\pi}$

c. $\pi,-\pi$

d. $2\pi,-2\pi$

TWO MARKS QUESTIONS (31-80)

31. Let S and T be two linear operators on R^3 defined by $S(x,y,z)=(x,x+y,x-y-z)$ $T(x,y,z)=(x+2z,y-z,x+y+z)$. Then

a. S is invertible but not T
b. T is invertible but not S
c. Both S and T are invertible
d. Neither S nor T is invertible

32. Let V, W and X be three finite dimensional vector spaces such that $\dim V=\dim X$. Suppose $S:V\rightarrow W$ and $T:W\rightarrow X$ are two linear maps such that to $S:V\rightarrow X$ is injective. Then

a. S and T are surjective
b. S is surjective and T is injective
c. S and T are injective
d. S is injective and T is surjective

33. If a square matrix of order 10 has exactly 4 distinct eigen values, then the degree of its minimal polynomial is

a. Least 4
b. At most 4
c. At least 6
d. At most 6

34. Consider the matrix $M = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$.

Then

a. M has no real eigen values
b. All real eigen values of M are positive
c. All real eigen values of M are negative
d. M has both positive and negative real eigen values

35. Consider the real inner product space $P[0,1]$ of all polynomials with the inner product $\langle f,g\rangle = \int_0^1 f(x)g(x)\,dx$. Let M = span {1}. The orthogonal projection of x^2 on to M is

a. 1
b. $\frac{1}{2}$
c. $\frac{1}{3}$
d. $\frac{1}{4}$

36. Let γ be a simple closed curve in the complex. Then the set of all possible values of $\oint_\gamma \frac{dz}{z(1-z^2)}$ is

a. $\{0, \pm\pi i\}$
b. $\{0, \pi i, 2\pi i\}$
c. $\{0, \pm\pi i, \pm 2\pi i\}$
d. $\{0\}$

37. The principal value of the improper integral $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2}dx$ is

a. $\frac{\pi}{e}$
b. πe
c. $\pi + e$
d. $\pi - e$

38. The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ that lie in the region $\left\{z \in C : 2 \le |z| < \frac{5}{2}\right\}$ is

a. 2
b. 3
c. 4
d. 5

39. Let $f:(0,2) \to R$ be defined by

$$f(x) = \begin{cases} x^2 & \text{If x is rational} \\ 2x-1 & \text{If x is irrational} \end{cases}$$

Then

a. f is differentiable exactly at one point
b. f is differentiable exactly at two points
c. f is not differentiable at any point in (0,2)
d. f is differentiable at every point in (0,2)

40. Let $f: R^2 \to R$ be defined by

$$f(x,y) = \begin{cases} x^2 + y^2 & \text{If x and y are rational} \\ 0 & \text{Otherwise} \end{cases}$$

Then

a. f is not continuous at (0,0)
b. f is continuous at (0,0) but not differentiable at (0,0)
c. f is differentiable only at (0,0)
d. f is differentiable every where

41. Let $f, g : R^2 \to R$ be defined by $f(x,y) = x^4 + y^2; g(x,y) = x^4 + y^2 - 10x^2 y$

Then at (0,0)

a. f has a local minimum but not g
b. g has a local minimum but not f
c. Both f and g have a local minimum
d. Neither f nor g has a local minimum

42. Suppose C_1 is the boundary of $\{(x,y) \in R^2 : 0 \le x \le 1, 0 \le y \le 1\}$ and C_2 is the boundary of $\{(x,y) \in R^2 : -1 \le x \le 0, -1 \le y \le 0\}$. Let

$$\alpha_i = \int_{C_i} xy^2 dx + (x^2 y + 2x) dy, \; i = 1,2$$

Be evaluated in the counterclockwise direction. Then

a. $\alpha_1 = 1, \alpha_2 = -1$
b. $\alpha_1 = \alpha_2 = 1$
c. $\alpha_1 = 2, \alpha_2 = -2$
d. $\alpha_1 = \alpha_2 = 2$

43. Consider R^2 with the usual metric and the functions

$f:[0,(2\pi) \to R^2$ and $g:[0,2\pi] \to R^2$

defined by

$f(t)=(\cos t,\sin t),\quad 0\le t<2\pi$ and $g(t)=(\cos t,\sin t),\ 0\le t\le 2\pi$.

Then on the respective domains

a. f is uniformly continuous but not g
b. g is uniformly continuous but not f
c. Both f and g are uniformly continuous
d. Neither f nor g is uniformly continuous

44. Let $f:R\to R$ be a nonzero function such that $|f(x)|\le\frac{1}{1+2x^2}$ for all $x\in R$. Define real valued functions f_n on R for all $n\in N$ by $f_n(x)=f(x+n)$. Then the series $\sum_{n=1}^{\infty}f_n(x)$ converges uniformly

a. On [0, 1] but not on [–1, 0]
b. On [–1,0] but not on [0,1]
c. On both [–1,0] and [0,1]
d. Neither on [–1,0] nor on [0,1]

45. Let E be a non measurable subset of (0,1). Define two functions f_1 and f_2 on (0,1) as follows:

$$f_1(x)=\begin{cases}1/x & if\quad x\in E\\ 0 & if\quad x\notin E\end{cases}$$ and $$f_2(x)=\begin{cases}0 & if\quad x\in E\\ 1/x & if\quad x\notin E\end{cases}.$$ Then

a. f_1 is measurable but not f_2
b. f_2 is measurable but not f_1
c. Both f_1 and f_2 are measurable
d. Neither f_1 nor f_2 is measurable

46. Consider the following improper integrals:

$$I_1=\int_1^{\infty}\frac{dx}{(1+x^2)^{1/2}}\ \text{and}\ I_2=\int_1^{\infty}\frac{dx}{(1+x^2)^{3/2}}$$

Then

a. I_1 converges but not I_2
b. I_2 converges but not I_1
c. Both I_1 and I_2 converge
d. Neither I_1 nor I_2 converges

47. A curve γ in the xy-plane is such that the line joining the origin to any point $P(x,y)$ on the curve and the line parallel to the y–axis through P are equally inclined to the tangent to the curve at P. Then, the differential equation of the curve γ is

a. $x\left(\frac{dy}{dx}\right)^2+2y\left(\frac{dy}{dx}\right)=x$
b. $x\left(\frac{dy}{dx}\right)^2+2y\left(\frac{dy}{dx}\right)=0$
c. $x\left(\frac{dx}{dy}\right)^2+2y\left(\frac{dx}{dy}\right)=0$
d. $x\left(\frac{dx}{dy}\right)^2+2y\left(\frac{dx}{dy}\right)=x$

48. Let $P_n(x)$ denote the Legendre polynomial of degree n. If

$$f(x)=\begin{cases}x, & -1\le x\le 0\\ 0, & 0\le x\le 1\end{cases}$$

And

$f(x)=a_0P_0(x)+a_1P_1(x)+a_2P_2(x)+.....,$

Then

a. $a_0=-\frac{1}{4},a_1=-\frac{1}{2}$
b. $a_0=-\frac{1}{4},a_1=\frac{1}{2}$
c. $a_0=\frac{1}{2},a_1=-\frac{1}{4}$
d. $a_0=-\frac{1}{2},a_1=-\frac{1}{4}$

49. If $J_n(x)$ and $Y_n(x)$ denote Bessel functions of order n of the first and the second kind, then the general solution of the differential equation $x\frac{d^2y}{dx^2}-\frac{dy}{dx}+xy=0$ is given by

a. $y(x)=\alpha xJ_1(x)+\beta xY_1(x)$
b. $y(x)=\alpha J_1(x)+\beta Y_1(x)$
c. $y(x)=\alpha J_0(x)+\beta Y_0(x)$
d. $y(x)=\alpha xJ_0(x)+\beta xY_0(x)$

50. The general solution of the system of differential equations

$$y+\frac{dz}{dx}=0$$
$$\frac{dy}{dx}-z=0$$

Is given by

a. $y=\alpha e^x+\beta e^{-x}$
$z=\alpha e^x-\beta e^{-x}$

b. $y=\alpha\cos x+\beta\sin x$
$z=\alpha\sin x-\beta\cos x$

c. $y=\alpha\sin x-\beta\cos x$
$z=\alpha\cos x+\beta\sin x$

d. $y=\alpha e^{x}-\beta e^{-x}$
$z=\alpha e^{x}+\beta e^{-x}$

51. It is required to find the solution of the differential equation
$$2x(2+x)\frac{d^2y}{dx^2}+2(3+x)\frac{dy}{dx}-xy=0$$
Around the point x = 0. The roots of the indicial equation are

a. $0,\frac{1}{2}$
b. 0,2
c. $\frac{1}{2},\frac{1}{2}$
d. $0,-\frac{1}{2}$

52. Consider the following statements.
S: Every non abelian group has a nontrivial abelian subgroup
T: Every nontrivial abelian group has a cyclic subgroup. Then

a. Both S and T are false
b. S is true and T is false
c. T is true and S is false
d. Both S and T are true

53. Let S_{10} denote the group of permutations on ten symbols $\{1,2,......,10\}$. The number of elements of S_{10} commuting with the element $\sigma=(13579)$ is

a. 5!
b. 5.5!
c. 5!5!
d. $\frac{10!}{5!}$

54. Match the following in an integral domain.
U. The only nilpotent element (s) a. 0
V. The only idempotent element (s) b. 1
W. The only unit and idempotent element (s) c. 0,1

a. $U-a;\ V-b;\ W-c$
b. $U-b; V-c; W-a$
c. $U-c;\ V-a;\ W-b$
d. $U-a; V-c; W-b$

55. Let Z be the ring of integers under the usual addition and multiplication. Then every nontrivial ring homomorphism $f:Z\to Z$ is

a. Both injective and surjective
b. Injective but not surjective
c. Surjective but not injective
d. Neither injective nor surjective

56. Let $X=C[0,1]$ be the space of all real valued continuous functions on $[0,1]$ Let $T:X\to R$ be a linear functional defined by $T(f)=f(1)$. Let $X_1=(X,\|.\|_1)$ and $X_2=(X,\|.\|_\infty)$. Then T is continuous

a. On X_1 but not X_2
b. On X_2 but not on X_1
c. On both X_1 and X_2
d. Neither on X_1 nor on X_2

57. Let $X=(C[0,1],\|.\|_p),1\le p\le\infty$ and if $f_n(t)\begin{cases} n(1-nt) & if\ \ 0\le t\le 1/n \\ 0 & if\ \ 1/n<t\le 1\end{cases}$
$S=\{f_n\in X:n>1\}$, then S is

a. Bounded if p = 1
b. Bounded if p = 2
c. Bounded if $p=\infty$
d. Unbounded for all p

58. Suppose the iterates x_n generated by $x_{n+1}=x_n-\frac{2f(x_n)}{f'(x_n)}$ where f' denotes the derivative of f, converges to a double zero $x=a$ of $f(x)$. Then the convergence has order

a. 1
b. 2
c. 3
d. 1.6

59. Suppose the matrix $M=\begin{pmatrix} 2 & \alpha & -1 \\ \alpha & 2 & 1 \\ -1 & 1 & 4\end{pmatrix}$ has a unique Cholesky decomposition of the form $M=LL^T$, where L is a lower triangular matrix. The range of values of α is

a. $-2<\alpha<2$
b. $\alpha>2$
c. $-2<\alpha<3/2$
d. $3/2<\alpha<2$

60. The Runge-Kutta method of order four is used to solve the differential equation $\frac{dy}{dx} = f(x), y(0) = 0$
With step size h. The solution at x = h is given by

a. $y(h) = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right]$

b. $y(h) = \frac{h}{6}\left[f(0) + 2f\left(\frac{h}{2}\right) + f(h)\right]$

c. $y(h) = \frac{h}{6}\left[f(0) + f(h)\right]$

d. $y(h) = \frac{h}{6}\left[2f(0) + f\left(\frac{h}{2}\right) + 2f(h)\right]$

61. The values of the constants α, β, x_1 for which the quadrature formula
$$\int_0^1 f(x)\,dx = \alpha f(0) + \beta f(x_1)$$
Is exact for polynomials of degree as high as possible, are

a. $\alpha = \frac{2}{3}, \beta = \frac{1}{4}, x_1 = \frac{3}{4}$

b. $\alpha = \frac{3}{4}, \beta = \frac{1}{4}, x_1 = \frac{2}{3}$

c. $\alpha = \frac{1}{4}, \beta = \frac{3}{4}, x_1 = \frac{2}{3}$

d. $\alpha = \frac{2}{3}, \beta = \frac{3}{4}, x_1 = \frac{1}{4}$

62. The partial differential equation
$$x\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial y} + y\frac{\partial u}{\partial x} = 0$$
is

a. Elliptic in the region $x < 0, y < 0, xy > 1$

b. Elliptic in the region $x > 0, y > 0, xy > 1$

c. Parabolic in the region $x < 0, y < 0, xy > 1$

d. Hyperbolic in the region $x < 0, y < 0, xy > 1$

63. A function $u(x,t)$, satisfies the wave equation
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$$
If $u\left(\frac{1}{2}, 0\right) = \frac{1}{4}, u\left(1, \frac{1}{2}\right) = 1$ and $u\left(0, \frac{1}{2}\right) = \frac{1}{2}$ then $u\left(\frac{1}{2}, 1\right)$ is

a. $\frac{7}{4}$

b. $\frac{5}{4}$

c. $\frac{4}{5}$

d. $\frac{4}{7}$

64. The Fourier transform $F(\omega)$ of $f(x). -\infty < x < \infty$ is defined by
$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-i\omega x}\,dx$$
The Fourier transform with respect to x of the solution $u(x,y)$ of the boundary value problem $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, -\infty < x < \infty, y > 0$
$u(x,0) = f(x), -\infty < x < \infty$ which remains bounded for large y is given by $U(\omega, y) = F(\omega) e^{-|\omega| y}$.
Then, the solution $u(x,y)$ is given by

a. $u(x,y) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(x-z)}{y^2+z^2}\,dz$

b. $u(x,y) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(x+z)}{y^2+z^2}\,dz$

c. $u(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(x-z)}{y^2+z^2}\,dz$

d. $u(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(x+z)}{y^2+z^2}\,dz$

65. It is required to solve the Laplace equation
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < a, 0 < y < b,$$
Satisfying the boundary conditions
$u(x,0) = 0,\ u(x,b) = 0,\ u(0,y) = 0$ and $u(a,y) = f(y)$.
If c_n's are constants, then the equation and the homogeneous boundary conditions determine the fundamental set of solutions of the form

a. $u(x,y)=\sum_{n=1}^{\infty} c_n \sin h\frac{n\pi x}{b}\sin\frac{n\pi y}{b}$

b. $u(x,y)=\sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{b}\sin\frac{n\pi y}{b}$

c. $u(x,y)=\sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{b}\sin h\frac{n\pi y}{b}$

d. $u(x,y)=\sum_{n=1}^{\infty} c_n \sin h\frac{n\pi x}{b}\sin h\frac{n\pi y}{b}$

66. Let the derivative of $f(t)$ with respect to time t be denoted by $\dot{f}$. If a Cartesian frame Oxy is in motion relative to a fixed Cartesian frame OXY specified by

$X = x\cos\theta + y\sin\theta$

$Y = -x\sin\theta + y\cos\theta$

Then the magnitude of the velocity v of a moving particle with respect to the OXY frame expressed in terms of the moving frame is given by

a. $v^2=\dot{x}^2+\dot{y}^2$

b. $v^2=\dot{x}^2+\dot{y}^2+\left(x^2+y^2\right)\dot{\theta}^2+2\dot{\theta}\left(\dot{x}y-\dot{y}x\right)$

c. $v^2=\dot{x}^2+\dot{y}^2+\dot{\theta}^2$

d. $v^2=\dot{x}^2+\dot{y}^2+\left(x^2+y^2\right)\dot{\theta}^2$

67. One end of an inextensible string of length a is connected to a mass M_1 lying on a horizontal table. The string passes through a small hole on the table and carries at the other end another mass M_2. If (r,θ) denotes the polar coordinates of the mass M_1 with respect to the hole as the origin in the plane of the table and g denotes the acceleration due to gravity, then the Lagrangian L of the system is given by

a. $L=\frac{1}{2}(M_1+M_2)\dot{r}^2+\frac{1}{2}M_1r^2\dot{\theta}^2-M_2g(r-a)$

b. $L=\frac{1}{2}M_1\dot{r}^2+\frac{1}{2}(M_1+M_2)r^2\dot{\theta}^2-M_2g(r-a)$

c. $L=\frac{1}{2}M_1\dot{r}^2+\frac{1}{2}(M_1+M_2)r^2\dot{\theta}^2-M_1g(r-a)$

d. $L=\frac{1}{2}(M_1+M_2)\dot{r}^2+\frac{1}{2}M_1r^2\dot{\theta}^2-M_1g(r-a)$

68. Consider R and S^1 with the usual topology where S^1 is the unit circle in R^2. Then

a. There is no continuous map from S^1 to R

b. Any continuous map from S^1 to R is the zero map

c. Any continuous map from S^1 to R is a constant map

d. There are non constant continuous map from S^1 to R

69. Consider R with the usual topology and R^ω, the countable product of R with product topology. If $D_n=[-n,n]\subseteq R$ and $f:R^\omega\to R$ is a continuous map, then $f\left(\prod_{n\in N}D_n\right)$ is of the form

a. $[a,b]$ for some $a\le b$

b. (a,b) for some $a<b$

c. Z

d. R

70. Let R^2 denote the plane with the usual topology and $U=\left\{(x,y)\in R^2 : xy<0\right\}$. Denote the number of connected components of U and $\bar{U}$ (the closure of U) by α and β respectively. Then

a. $\alpha=\beta=1$

b. $\alpha=1,\beta=2$

c. $\alpha=2,\beta=1$

d. $\alpha=\beta=2$

71. Under the usual topology on R^3, the map $f:R^3\to R^3$ defined by $f(x,y,z)=(x+1,y-1,z)$ is

a. Neither open nor closed

b. Open but not closed

c. Both open and closed

d. Closed but not open

72. Let X_1,X_2,X_3 be a random sample of size 3 chosen from a population with probability distribution $P(X=1)=P$ and $P(X=0)=1-p=q,\quad 0<p<1$. The sampling distribution $f(.)$ of the statistic $Y=\max[X_1,X_2,X_3]$ is

a. $f(0) = p^3; f(1) = 1 - p^3$

b. $f(0) = q; f(1) = p$

c. $f(0) = q^3; f(1) = 1 - q^3$

d. $f(0) = p^3 + q^3; f(1) = 1 - p^3 - q^3$

73. Let $\{X_n\}$ be a sequence of independent random variables with

$p(X_n = n^\alpha) = p(X_n = -n^\alpha) = \frac{1}{2}$.

The sequence $\{X_n\}$ obeys the weak law of large numbers if

a. $\alpha < \frac{1}{2}$

b. $\alpha = \frac{1}{2}$

c. $\frac{1}{2} < \alpha \le 1$

d. $\alpha > 1$

74. Let X be a random variable with $P(X = 1) = P$ and $p(X = 0) = 1 - p = q,\ 0 < p < 1$. If μ_n denotes the n[th] moment about the mean, then $\mu_{2n+1} = 0$ if and only if

a. $p = \frac{1}{4}$

b. $p = \frac{1}{3}$

c. $p = \frac{2}{3}$

d. $p = \frac{1}{2}$

75. Consider the following primal Linear Programming Problem (LPP).

Maximize $z = 3x_1 + 2x_2$

Subject to $x_1 - x_2 \le 1$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

The dual of this problem has

a. Infeasible optimal solution

b. Unbounded optimal objective value

c. A unique optimal solution

d. Infinitely many optimal solutions

76. The cost matrix of a transportation problem is given by

6	4	1	5
3	9	2	7
4	3	6	2

The following values of the basis variables were obtained at the first iteration: $x_{11} = 6,\ x_{12} = 8, x_{22} = 2, x_{23} = 14, x_{33} = 4$

Then

a. The current solution is optimal

b. The current solution is non optimal and the entering and leaving variables are x_{31} and x_{33} respectively

c. The current solution is non optimal and the entering and leaving variables are x_{21} and x_{12} respectively

d. The current solution is non optimal and the entering and leaving variables are x_{14} and x_{12} respectively

77. In a balanced transportation problem, if all the unit transportation costs c_{ij} are decreased by a nonzero constant α, then in optimal solution of the revised problem

a. The values of the decision variables and the objective value remain unchanged

b. The values of the decision variables change but the objective value remains unchanged

c. The values of the decision variables remain unchanged but the objective value changes

d. The values of the decision variables and the objective value change

78. Consider the following linear programming problem (LPP).

Maximize $z = 3x_1 + x_2$

Subject to $x_1 + 2x_2 \le 5$

$x_1 + x_2 - x_3 \le 2$

$7x_1 + 3x_2 - 5x_3 \le 20$

$x_1, x_2, x_3 \ge 0$

The nature of the optimal solution to the problem is

a. Non degenerate alternative optima

b. Degenerate alternative optima

c. Degenerate unique optimal

d. Non degenerate unique optimal

79. The extremum of the functional

$$I = \int_0^1 \left[\left(\frac{dy}{dx} \right)^2 + 12xy \right] dx$$

Satisfying the conditions $y(0)=0$ and $y(1)=1$ is attained on the curve

a. $y=\sin^2\frac{\pi x}{2}$

b. $y=\sin\frac{\pi x}{2}$

c. $y=x^3$

d. $y=\frac{1}{2}\left[x^3+\sin\frac{\pi x}{2}\right]$

80. The integral equation

$$y(x)=x-\int_0^x (x-t)y(t)\,dt$$

Is solved by the method of successive approximations. Starting with initial approximation $y(x)=x$ the second approximation $y_2(x)$ is given by

a. $y_2(x)=x+\frac{x^3}{3!}+\frac{x^5}{5!}$

b. $y_2(x)=x+\frac{x}{3!}$

c. $y_2(x)=x-\frac{x^3}{3!}$

d. $y_2(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}$

Linked answer questions: Q. 81a to Q. 85b carry two marks each.

Statement for linked answer Questions 81a and 81b:

Let V be the vector space of real polynomials of degree at most 2. Define a linear operator $T:V\to V$

$$T\left(x^i\right)=\sum_{j=0}^{i} x^j, i=0,1,2$$

81a. The matrix of T^{-1} with respect to the basis $\{1,x,x^2\}$ is

(a) $\begin{pmatrix}1&1&1\\1&1&0\\1&0&0\end{pmatrix}$

(b) $\begin{pmatrix}1&-1&0\\0&1&-1\\0&0&1\end{pmatrix}$

(c) $\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$

(d) $\begin{pmatrix}1&0&0\\-1&1&0\\0&-1&1\end{pmatrix}$

81b. The dimension of the eigen space of T^{-1} corresponding to the eigen value 1 is

(a) 4

(b) 3

(c) 2

(d) 1

Statement for Linked answer Questions 81a and 82b

Let H be a real Hilbert space, $p\in H$, $p\neq 0$ and $G=\left|x\in H:\langle x,p\rangle=0\right|$ and $q\in H/G$.

82a. The orthogonal projection of q onto G is

(a) $q-\frac{\langle q,p\rangle}{\|p\|^2}$

(b) $q-\frac{\langle q,p\rangle p}{\|p\|}$

(c) $q-\langle q,p\rangle p$

(d) $q-\langle q,p\rangle\|p\|p$

82b. In particular, if $H=L_2[0,1], G=\left\{f\in L_2[0,1]:\int_0^1 xf(x)\,dx=0\right\}$ and $q=x^2$, then the orthogonal of q onto G is

(a) $x^2-\frac{3}{4}x$

(b) x^2-3x

(c) $x^2-\frac{3}{5}x$

(d) $x^2-\frac{3}{2}x$

Statement for Linked Answer Questions 83a and 83b

It is required to solve the system SX = T, where

$S=\begin{pmatrix}2&-1&1\\2&2&2\\-1&-1&2\end{pmatrix}$, $T=\begin{pmatrix}-1\\4\\-5\end{pmatrix}$ by the Gauss-Seidel iteration onethod.

83a Suppose S is written in the form S = M–L–U, where, M is a diagonal matrix, L is a strictly lower triangular matrix and U is a strictly upper triangular matrix. If the iteration process is expressed as $X_{n+1}=QX_n+F$, the Q is given by

(a) $Q=(M+L)^{-1}U$

(b) $Q=M^{-1}(L+U)$

(c) $Q=(M-L)^{-1}U$

(d) $Q=M^{-1}(L-U)$

83b. The matrix Q is given by

(a) $\begin{pmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}$

(b) $Q=\begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

(c) $Q=\begin{pmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$

(d) $Q=\begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Statement for Linked Answer Questions 84a and 84b

Consider the one dimensional heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<1,\ t>0$ with the initial condition $u(x,0)=2\cos^2 \pi x$ and the boundary conditions. $\frac{\partial u}{\partial t}(0,t)=0=\frac{\partial u}{\partial t}(1,t)$.

84a. The temperature $u(x,t)$ is given by

(a) $u(x,t)=1-e^{-4\pi^2 t}\cos 2\pi x$

(b) $u(x,t)=1+e^{-4\pi^2 t}\cos 2\pi x$

(c) $u(x,t)=1-e^{-4\pi^2 t}\sin 2\pi x$

(d) $u(x,t)=1+e^{-4\pi^2 t}\sin 2\pi x$

84b. The heat flux F at $\left(\frac{1}{4},t\right)$ is given by

(a) $F=2\pi e^{-4\pi^2 t}$

(b) $F=-2\pi e^{-4\pi^2 t}$

(c) $F=4\pi e^{-2\pi^2 t}$

(d) $F=-4\pi e^{-2\pi^2 t}$

Statement for Linked Answer Questions 85a and 85b

Let the random variables X and Y be independent Poisson variates with parameters λ_1 and λ_2 respectively.

85a. The conditional distribution of X given X+Y is

(a) Poisson

(b) Hyper geometric

(c) Geometric

(d) Binomial

85b. The regression equation of X on X+Y is given by

(a) $E(X \mid X+Y)=XY\frac{\lambda_1}{\lambda_1+\lambda_2}$

(b) $E(X \mid X+Y)=(X+Y)\frac{\lambda_1}{\lambda_1+\lambda_2}$

(c) $E(X \mid X+Y)=(X+Y)\frac{\lambda_1}{\lambda_1+\lambda_2}$

(d) $E(X \mid X+Y)=XY\frac{\lambda_1}{\lambda_1+\lambda_2}$