MATHEMATICS

ONE MARKS QUESTIONS (1-20)

1. The dimension of the vector space $V=\left\{A=\left(a_{ij}\right)_{n\times n};a_{ij}\in C,a_{ij}=-a_{ji}\right\}$ over field R is
   (a.) n^2
   (b.) n^2-1
   (c.) n^2-n
   (d.) $\frac{n^2}{2}$

2. The minimal polynomial associated with the matrix $\begin{bmatrix}0&0&3\\1&0&2\\0&1&1\end{bmatrix}$ is
   (a.) x^3-x^2-2x-3
   (b.) x^3-x^2+2x-3
   (c.) x^3-x^2-3x-3
   (d.) x^3-x^2+3x-3

3. For the function $f(z)=\sin\left(\frac{1}{\cos(1/z)}\right)$, the point z = 0 is
   (a.) a removable singularity
   (b.) a pole
   (c.) an essential singularity
   (d.) a non-isolated singularity

4. Let $f(z)=\sum_{n=0}^{15}z^n$ *for* $z\in C$. If $C:|z-i|=2$ then $\oint_C\frac{f(z)\,dz}{(z-i)^{15}}=$
   (a.) $2\pi i(1+15i)$
   (b.) $2\pi i(1-15i)$
   (c.) $4\pi i(1+15i)$
   (d.) $2\pi i$

5. For what values of α and β, the quadrature formula $\int_{-1}^{1}f(x)\,dx\approx\alpha f(-1)+f(\beta)$ is exact for all polynomials of degree ≤ 1?
   (a.) $\alpha=1,\beta=1$
   (b.) $\alpha=-1,\beta=1$
   (c.) $\alpha=1,\beta=-1$
   (d.) $\alpha=-1,\beta=-1$

6. Let $f:[0,4]\to R$ be a three times continuously differentiable function. Then the value of $f[1,2,3,4]$ is
   (a.) $\frac{f''(\xi)}{3}$ for some $\xi\in(0,4)$
   (b.) $\frac{f''(\xi)}{6}$ for some $\xi\in(0,4)$
   (c.) $\frac{f'''(\xi)}{3}$ for some $\xi\in(0,4)$
   (d.) $\frac{f'''(\xi)}{6}$ for some $\xi\in(0,4)$

7. Which one of the following is TRUE?
   (a.) Every linear programming problem has a feasible solution.
   (b.) If a linear programming problem has an optimal solution then it is unique.
   (c.) The union of two convex sets is necessarily convex.
   (d.) Extreme points of the disk $x^2+y^2\le 1$ are the point on the circle $x^2+y^2=1$.

8. The dual of the linear programming problem:
   Minimize c^Tx subject to $Ax\ge b$ and $x\ge 0$ is
   (a.) Maximize b^Tw subject to $A^Tw\ge c$ and $w\ge 0$
   (b.) Maximize b^Tw subject to $A^Tw\le c$ and $w\ge 0$
   (c.) Maximize b^Tw subject to $A^Tw\le c$ and w is unrestricted
   (d.) Maximize b^Tw subject to $A^Tw\ge c$ and w is unrestricted

9. The resolvent kernel for the integral equation $u(x)=F(x)+\int_{\log 2}^{x}f^{(t-x)}u(t)\,dt$ is
   (a.) $\cos(x-t)$
   (b.) 1
   (c.) e^{t-x}

(d.) $e^{2(t-x)}$

10. Consider the metrics $d_2(f,g)=\left(\int_a^b |f(t)-g(t)|^2\,dt\right)^{1/2}$ and $d_\infty(f,g)=\sup_{t\in[a,b]}|f(t)-g(t)|$ on the space $X=C[a,b]$ of all real valued continuous functions on $[a,b]$. Then which of the following is TRUE?
(a.) Both (X,d_2) and (X,d_∞) are complete.
(b.) (X,d_2) is complete but (X,d_∞) is NOT complete.
(c.) (X,d_∞) is complete but (X,d_2) is NOT complete.
(d.) Both (X,d_2) and (X,d_∞) are NOT complete.

11. A function $f:R\to R$ need NOT be Lebesgue measurable if
(a.) f is monotone
(b.) $\{x\in R: f(x)\geq\alpha\}$ is measurable for each $\alpha\in R$
(c.) $\{x\in R: f(x)=\alpha\}$ is measurable for each $\alpha\in R$
(d.) For each open set G in R, $f^{-1}(G)$ is measurable

12. Let $\{e_n\}_{n=1}^{\infty}$ be an orthonormal sequence in a Hilbert space H and let $x(\neq 0)\in H$. Then
(a.) $\lim_{n\to\infty}\langle x,e_n\rangle$ does not exist
(b.) $\lim_{n\to\infty}\langle x,e_n\rangle=\|x\|$
(c.) $\lim_{n\to\infty}\langle x,e_n\rangle=1$
(d.) $\lim_{n\to\infty}\langle x,e_n\rangle=0$

13. The subspace $Q\times[0,1]$ of R^2 (with the usual topology) is
(a.) dense is R^2
(b.) connected
(c.) separable
(d.) compact

14. $Z_2[x]/\langle x^3+x^2+1\rangle$ is
(a.) a field having 8 elements
(b.) a field having 9 elements
(c.) an infinite field
(d.) NOT a field

15. The number of element of a principal ideal domain can be
(a.) 15
(b.) 25
(c.) 35
(d.) 36

16. Let, F, G and H be pair wise independent events such that $P(F)=P(G)=P(H)=\frac{1}{3}$ and $(F\cap G\cap H)=\frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is
(a.) $\frac{11}{12}$
(b.) $\frac{7}{12}$
(c.) $\frac{5}{12}$
(d.) $\frac{3}{4}$

17. Let X be a random variable such that $E(X^2)=E(X)=1$. Then $E(X^{100})$ =
(a.) 0
(b.) 1
(c.) 2^{100}
(d.) $2^{100}+1$

18. For which of the following distributions, the weak law of large numbers does NOT hold?
(a.) Normal
(b.) Gamma
(c.) Beta
(d.) Cauchy

19. If $D\equiv\frac{d}{dx}$ then the value of $\frac{1}{(xD+1)}(x^{-1})$ is
(a.) $\log x$
(b.) $\frac{\log x}{x}$
(c.) $\frac{\log x}{x^2}$
(d.) $\frac{\log x}{x^3}$

20. The equation

$$\left(\alpha xy^3 + y\cos x\right)dx + \left(x^2y^2 + \beta \sin x\right)dy = 0$$

is exact for

(a.) $\alpha = \frac{3}{2}, \beta = 1$

(b.) $\alpha = 1, \beta = \frac{3}{2}$

(c.) $\alpha = \frac{2}{3}, \beta = 1$

(d.) $\alpha = 1, \beta = \frac{2}{3}$

TWO MARKS QUESTIONS (21-60)

21. If $A = \begin{pmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{pmatrix}$, then the trace of A^{102} is

(a.) 0
(b.) 1
(c.) 2
(d.) 3

22. Which of the following matrices is NOT diagonalizable?

(a.) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

(b.) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$

(c.) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(d.) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

23. Let V be the column space of the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$. Then the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ on V is

(a.) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(b.) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(c.) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(d.) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

24. Let $\sum_{n=-\infty}^{\infty} a_n (z+1)^n$ be the Laurent series expansion of $f(z) = \sin\left(\frac{z}{z+1}\right)$. Then $a_{-2} =$

(a.) 1
(b.) 0
(c.) $\cos(1)$
(d.) $\frac{-1}{2}\sin(1)$

25. Let $u(x,y)$ be the real part of an entire function $f(z) = u(x,y) + iv(x,y)$ for $z = x + iy \in C$. If C is the positively oriented boundary of a rectangular region R in R^2, then $\oint_C \left[\frac{\partial u}{\partial y}dx - \frac{\partial u}{\partial x}dy\right] =$

(a.) 1
(b.) 0
(c.) 2π
(d.) π

26. Let $\phi : [0,1] \to R$ be three times continuously differentiable, Suppose that the iterates defined by $x_{n+1} = \phi(x_n), n \geq 0$ converge to the fixed point ξ of ϕ. If the order of convergence is three then

(a.) $\phi'(\xi) = 0, \phi''(\xi) = 0$

(b.) $\phi'(\xi) \neq 0, \phi''(\xi) = 0$

(c.) $\phi'(\xi) = 0, \phi''(\xi) \neq 0$

(d.) $\phi'(\xi) \neq 0, \phi''(\xi) \neq 0$

27. Let $f:[0,2]\to R$ be a twice continuously differentiable function. If $\int_0^2 f(x)dx \approx 2f(1)$, then the error in the approximation is

(a.) $\frac{f'(\xi)}{12}$ for some $\xi\in(0,2)$

(b.) $\frac{f'(\xi)}{2}$ for some $\xi\in(0,2)$

(c.) $\frac{f''(\xi)}{3}$ for some $\xi\in(0,2)$

(d.) $\frac{f''(\xi)}{6}$ for some $\xi\in(0,2)$

28. For a fixed $t\in R$, consider the linear programming problem:

Maximize $z=3x+4y$

Subject to $x+y\le 100$

$x+3y\le t$

and $x\ge 0, y\ge 0$

The maximum value of z is 400 for t =

(a.) 50
(b.) 100
(c.) 200
(d.) 300

29. The minimum value of

$z=2x_1-x_2+x_3-5x_4+22x_5$ subject to

$x_1-2x_4+x_5=6$

$x_2+x_4-4x_5=3$

$x_3+3x_4+2x_5=10$

$x_j\ge 0, j=1,2,....,5$

is

(a.) 28
(b.) 19
(c.) 10
(d.) 9

30. Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

5	23	14	8
10	25	1	23
35	16	15	12
16	23	11	7

is

(a.) 29
(b.) 52
(c.) 26
(d.) 44

31. Which of the following sequence $\{f_n\}_{n=1}^{\infty}$ of functions does NOT converge uniformly on [0, 1]?

(a.) $f_n(x)=\frac{e^{-x}}{n}$

(b.) $f_n(x)=(1-x)^n$

(c.) $f_n(x)=\frac{x^2+nx}{n}$

(d.) $f_n(x)=\frac{\sin(nx+n)}{n}$

32. Let $E=\{(x,y)\in R^2:0<x<y\}$.Then $\iint_E ye^{-(x+y)}dxdy=$

(a.) $\frac{1}{4}$

(b.) $\frac{3}{2}$

(c.) $\frac{4}{3}$

(d.) $\frac{3}{4}$

33. Let $f_n(x)=\frac{1}{n}\sum_{k=0}^{n}\sqrt{k(n-k)}\binom{n}{k}x^k(1-x)^{n-k}$ for $x\in[0,1], n=1,2,....$ If $\lim_{n\to\infty} f_n(x)=f(x)$ for $x\in[0,1]$, then the maximum value of $f(x)$ on [0,1] is

(a.) 1

(b.) $\frac{1}{2}$

(c.) $\frac{1}{3}$

(d.) $\frac{1}{4}$

34. Let $f:(c_{00},\|.\|_1)\to C$ be a non zero continuous linear functional. The number of Hahn-Banach extensions of f to $(l^1,\|.\|_1)$ is

(a.) One
(b.) Two
(c.) Three
(d.) infinite

35. If $I:\left(l^1,\|\cdot\|_2\right)\to\left(l^1,\|\cdot\|_1\right)$ is the identity map, then
(a.) Both I and I^{-1} are continuous
(b.) I is continuous but I^{-1} is NOT continuous
(c.) I^{-1} is continuous but I is NOT continuous
(d.) Neither I and I^{-1} is continuous

36. Consider the topology $\tau=\{G\subseteq R : R\setminus G$ is compact in $(R,\tau_u)\}\cup\{\phi,R\}$ on R, where τ_u is the usual topology on R and ϕ is the empty set. Then (R,τ) is
(a.) a connected Hausdorff space
(b.) connected but NOT Hausdorff
(c.) hausdorff but NOT connected
(d.) neither connected nor Hausdorff

37. Let
$\tau_1=\{G\subseteq R : G$ is finite or $R\setminus$ G is finite$\}$
and
$\tau_2=\{G\subseteq R : G$ is contable or $R\setminus$ G is contable$\}$
Then
(a.) neither τ_1 nor τ_2 is a topology on R
(b.) τ_1 is a topology on R but τ_2 is NOT a topology on R
(c.) τ_2 is a topology on R but τ_1 is NOT a topology on R
(d.) both τ_1 and τ_2 are topologies on R

38. Which one of the following ideals of the ring $Z[i]$ of Gaussian integers is NOT maximal?
(a.) $\langle 1+i\rangle$
(b.) $\langle 1-i\rangle$
(c.) $\langle 2+i\rangle$
(d.) $\langle 3+i\rangle$

39. If Z(G) denotes the centre of a group G, then the order of the quotient group G/Z(G) cannot be
(a.) 4
(b.) 6
(c.) 15
(d.) 25

40. Let Aut(G) denote the group of automorphism of a group G. Which one of the following is NOT a cyclic group?
(a.) $Aut(Z_4)$
(b.) $Aut(Z_6)$
(c.) $Aut(Z_8)$
(d.) $Aut(Z_{10})$

41. Let X be a non-negative integer valued random variable with $E(X^2)=3$ and $E(X)=1$. Then $\sum_{i=1}^{\infty} iP(X\ge i)=$
(a.) 1
(b.) 2
(c.) 3
(d.) 4

42. Let X be a random variable with probability density function $f\in\{f_0,f_1\}$, where
$$f_0(x)=\begin{cases}2x, & if\ 0<x<1\\ 0, & otherwise\end{cases}$$ and
$$f_1(x)=\begin{cases}3x^2, & if\ 0<x<1\\ 0, & otherwise\end{cases}$$
For testing the null hypothesis $H_0: f\equiv f_0$ against the alternative hypothesis $H_1: f\equiv f_1$ at level of significance α = 0.19, the power of the most powerful test is
(a.) 0.729
(b.) 0.271
(c.) 0.615
(d.) 0.385

43. Let X and Y be independent and identically distributed U(0, 1) random variables. Then $P\left(Y<\left(X-\frac{1}{2}\right)^2\right)=$
(a.) $\frac{1}{12}$
(b.) $\frac{1}{4}$
(c.) $\frac{1}{3}$
(d.) $\frac{2}{3}$

44. Let X and Y be Banach spaces and let $T:X\to Y$ be a linear map. Consider the statements:
P: If $x_n\to x$ in X then $Tx_n\to Tx$ in Y.

Q: If $x_n \to x$ in X and $Tx_n \to y$ in Y then $Tx = y$.

Then

(a.) P implies Q and Q implies P
(b.) P implies Q but Q does not imply P
(c.) Q implies P but P does not imply Q
(d.) Neither P implies Q nor Q implies P

45. If $y(x) = x$ is a solution of the differential equation $y'' - \left(\frac{2}{x^2} + \frac{1}{x}\right)(xy' - y) = 0, 0 < x < \infty$, then its general solution is

(a.) $(\alpha + \beta e^{-2x})x$
(b.) $(\alpha + \beta e^{2x})x$
(c.) $\alpha x + \beta e^x$
(d.) $(\alpha e^x + \beta)x$

46. Let $P_n(x)$ be the Legendra polynomial of degree n such that $P_n(1) = 1, n = 1, 2, ...$ If $\int_{-1}^{1}\left(\sum_{j=1}^{n}\sqrt{j(2j+1)}P_j(x)\right)^2 dx = 20$, then n =

(a.) 2
(b.) 3
(c.) 4
(d.) 5

47. The integral surface satisfying the equation $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x^2 + y^2$ and passing through the curve $x = 1 - 1, y = 1 + t, z = 1 + t^2$ is

(a.) $z = xy + \frac{1}{2}(x^2 - y^2)^2$
(b.) $z = xy + \frac{1}{4}(x^2 - y^2)^2$
(c.) $z = xy + \frac{1}{8}(x^2 - y^2)^2$
(d.) $z = xy + \frac{1}{16}(x^2 - y^2)^2$

48. For the diffusion problem $u_{xx} = u_t (0 < x < \pi, t > 0)$, $u(0,t) = 0$, $u(\pi,t) = 0$ and $u(x,0) = 3\sin 2x$, the solution is given by

(a.) $3e^{-t}\sin 2x$
(b.) $3e^{-4t}\sin 2x$
(c.) $3e^{-9t}\sin 2x$
(d.) $3e^{-2t}\sin 2x$

49. A simple pendulum, consisting of a bob of mass m connected with a string of length a, is oscillating in a vertical plane. If the string is making an angle θ with the vertical, then the expression for the Lagrangian is given as

(a.) $ma^2\left(\theta^2 - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$
(b.) $2mga\sin^2\left(\frac{\theta}{2}\right)$
(c.) $ma^2\left(\frac{\theta^2}{2} - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$
(d.) $\frac{ma}{2}\left(\theta^2 - \frac{2g}{a}\cos\theta\right)$

50. The extremal of the functional $\int_0^1\left(y + x^2 + \frac{y'^2}{4}\right)dx, y(0) = 0, y(1) = 0$ is

(a.) $4(x^2 - x)$
(b.) $3(x^2 - x)$
(c.) $2(x^2 - x)$
(d.) $x^2 - x$

Common Data for Questions (51 & 52)

Let $T: R^3 \to R^3$ be the linear transformation defined by
$T(x_1, x_2, x_3) = (x_1 + 3x_2 + 2x_3 + 3x_1 + 4x_2 + x_3, 2x_1 + x_2 - x_3)$

51. The dimension of the range space of T^2 is

(a.) 0
(b.) 1
(c.) 2
(d.) 3

52. The dimension of the null space of T^3 is

(a.) 0
(b.) 1
(c.) 2
(d.) 3

Common Data for Questions (53 & 54)

Let $y_1(x) = 1 + x$ and $y_2(x) = e^x$ be two solutions of $y''(x) + P(x)y'(x) + Q(x)y(x) = 0$.

53. $P(x) =$

(a.) $1 + x$

(b.) $-1-x$

(c.) $\frac{1+x}{x}$

(d.) $\frac{-1-x}{x}$

54. The set of initial conditions for which the above differential equation has NO solution is

(a.) $y(0)=2, y'(0)=1$

(b.) $y(1)=0, y'(1)=1$

(c.) $y(1)=1, y'(1)=0$

(d.) $y(2)=1, y'(2)=2$

Common Data for Questions (55 & 56)

Let X and Y be random variables having the joing probability density function

$$f(x,y)=\begin{cases}\frac{1}{\sqrt{2\pi y}}e^{\frac{-1}{2y}(x-y)^2}, & if\ -\infty<x<\infty, 0<y<1\\ 0, & otherwise\end{cases}$$

55. The variance of the random variable X is

(a.) $\frac{1}{12}$

(b.) $\frac{1}{4}$

(c.) $\frac{7}{12}$

(d.) $\frac{5}{12}$

56. The covariance between the random variables X and Y is

(a.) $\frac{1}{3}$

(b.) $\frac{1}{4}$

(c.) $\frac{1}{6}$

(d.) $\frac{1}{12}$

Statement for Linked Answer Question (57 and 58)

Consider the function $f(z)=\frac{e^{iz}}{z(z^2+1)}$.

57. The residue of f at the isolated singular point in the upper half plane $\{z=x+iy\in C: y>0\}$ is

(a.) $\frac{-1}{2e}$

(b.) $\frac{-1}{e}$

(c.) $\frac{e}{2}$

(d.) 2

58. The Cauchy Principal Value of the integral $\int_{-\infty}^{\infty}\frac{\sin x dx}{x(x^2+1)}$ is

(a.) $-2\pi(1+2e^{-1})$

(b.) $\pi(1+e^{-1})$

(c.) $2\pi(1+e)$

(d.) $-\pi(1+e^{-1})$

Statement for Linked Answer Question (59 and 60)

Let $f(x,y)=kxy-x^3y-xy^3$ for $(x,y)\in R^2$, where k is a real constant. The directional derivative of f at the point (1, 2) in the direction of the unit vector $u=\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ is $\frac{15}{\sqrt{2}}$.

59. The value of k is

(a.) 2

(b.) 4

(c.) 1

(d.) -2

60. The value of f at a local minimum in the rectangular region $R=\left\{(x,y)\in R^2: |x|<\frac{3}{2}, |y|<\frac{3}{2}\right\}$ is

(a.) - 2

(b.) - 3

(c.) $\frac{-7}{8}$

(d.) 0