Part-B

1. Let A be a real symmetric n × n matrix. Then a necessary and sufficient condition for the map $(x, y) \to x^T Ay$ to be an inner product on $\mathbb{R}^n$ (considered as the set of column vectors) is

   1. A is non-singular.
   2. A is positive definite.
   3. A is positive semi-definite.
   4. A is identity matrix.

2. The set of points where f(x) = |sin x| is not differentiable is

   1. empty
   2. {0}
   3. $\{k\pi : k \in \mathbb{Z}\}$
   4. $\{k\pi / 2 : k \in \mathbb{Z}\}$

3. Let f(x) $\pm x^3 - \sin x, x \in [-\pi/2, \pi/2]$. Then f is

   1. bounded and of bounded variation.
   2. unbounded and of bounded variation.
   3. unbounded but not of bounded variation.
   4. bounded but not off bounded variation.

4. Let $S = \{(x, y, z) \in \mathbb{R}^3 \mid ax^2 + by^2 + cz^2 = 1\}$ where a, b, c are all positive. Then the set S is

   1. connected but not compact in $\mathbb{R}^3$.
   2. not connected but compact in $\mathbb{R}^3$.

3. connected and compact in ¡ ^{3}.

4. not connected and not compact in ¡ ^{3}.

5. Let $f_n : [0,1] \to$ ¡ be defined by $f_n(x) = \begin{cases} 1-n^2x^2 & 0 \le |x| \le 1/n \\ 0 & 1/n \le |x| 1 \end{cases}$ *for* $n =$ 1,2,………

Then the sequence of functions f_n

1. converges point wise but not uniformly
2. converges uniformly
3. converges point wise and the limit function is continuous
4. does not converge pointwise.

6. Let $f_n : [0,1] \to$ ¡ be defined by $f_n(x) = nx^{n-1}$ for n = 1, 1, 2,…..

Then

1. lim inf $\int_0^1 f_n(x)\,dx = \int_0^1 \liminf\ f_n(x)\,dx$
2. $\int_0^1 f_n(x)\,dx \ne \int_0^1 \liminf -\ f_n(x)\,dx$
3. lim sup $\int_0^1 f_n(x)\,dx = \int_0^1 \liminf\ f_n(x)\,dx$
4. lim sum $\int_0^1 f_n(x)\,dx = \int_0^1 \limsup f_n(x)\,dx$

7. Let T : $e^2 \to e^2$ be defined by

$T\left(x_n\right)=\left(\left(\frac{1}{2}\right)^n x_n\right)$. Then

1. ||T|| = 1
2. T is not bounded
3. ||T|| = 0
4. ||T|| = 1/2

8. The value of $\int_C \frac{dz}{z^2+4x+3} where C(t) \setminus 2e^{it}, 0 \le t \le 2\pi$ is.

1. 0
2. πi
3. 2πi
4. 4πi

9. Let f : £ → £ be a non-constant entire function. Then f(z) = cos z for all z in £ if f(x) = cos z on D where D is

1. {z = x + iy| x is rational }
2. {z = x + iy | x is an integer}
3. {z = x + iy | x and y are integers}
4. {z = (2n+1)π | n ∈ Z}

10. Let f : ¡ ^{2} → ¡ ^{3} be defined by f(u, v) = (u, v, u + v). Then

1. f is differentiable at (1, 2) and the derivative, as a linear map is given by (h, k) →(h, k, h+k)
2. f is not differentiable at (1, 2)

3. f is differentiable at (1, 2) and the derivative as a linear map is given by $(h, k) \to (h, 2k, h+2k)$

4. f is differentiable at (1, 2) and the derivative as a linear map is given by $(h, k) \to (1, 2, 3)$

5. Let C[0, 1] be the space of all continuous functions with sup norm. Let E denote the subspace of all polynomials in C[0, 1]. Then

   1. E is closed and infinite dimensional
   2. E is closed and finite dimensional
   3. E is not closed and infinite dimensional
   4. E is not closed and finite dimensional

6. A homogeneous system of 5 linear equations in 6 variables admits

   1. no solution in $\mathbb{R}^6$
   2. a unique solution in $\mathbb{R}^6$
   3. infinitely many solutions in R
   4. finite, but more than 2 solutions in $\mathbb{R}^6$

7. Let V be the space of all linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^2$ under usual addition and scalar multiplication. Then

   1. V is a vector space of dimension 5.
   2. V is a vector space of dimension 6.
   3. V is a vector space of dimension 8.
   4. V is a vector space of dimension 9.

8. Let R be a commutative ring. Let I and J be ideals of R.

Let I – J = {x–y|x$\in I, y \in J$ } and

IJ = $\{xy \mid x, \in I, y \in J\}$. Then.

1. I–J is an ideal and IJ is an ideal in R
2. I–J is an ideal and IJ need not be an ideal in R.
3. I–J need not be an ideal but IJ is an ideal in R.
4. Neither I–J nor IJ need to be an ideal in R.

9. Let $V = \left\{ \left(x_{1,}, \ldots, x_{100} \right) \in \mathbb{R}^{100} \mid x_1 2x_2 = 3x_3 \text{ and } x_{51-}x_{52} \ldots x_{100} = 0 \right\}$ Then

1. dim V = 98	2. dim V = 49
3. dim V = 99	4. dim V = 97

10. Let A and B be finite sets of m and n elements respectively, with m < n. Then there is a

1. bijection from A onto B
2. Surjective function from A onto B
3. One-to-one (injective) function from A to B and a surjective function from B to A.
4. surjective function from A to B and an injective function from B to A.

11. Consider an m × m matrix

$$A = \begin{bmatrix} a & b & b & \ldots & b \\ b & a & b & \ldots & . \\ \vdots & & & & \\ b & b & \ldots & b & a \end{bmatrix}$$

A is nonsingular if and only if

1. $a \neq b$

2. $a \neq -(m-1)b$

3. $a \neq b$ or $a \neq -(m-1)b$

4. $a \neq b$ and a $a \neq -(m-1)b$

12. Let A : $\mathbb{R}^6 \to \mathbb{R}^5$ *and* $B : \mathbb{R}^5 \to \mathbb{R}^7$ be two linear transformation. The which of the following can be true.

1. A and B are one-one

2. A is one-one and B is not one-one.

3. A is onto and B is one-one

4. A and B both are onto.

13. Let A be an m × n a matrix with rank m and B be an p × m a matrix with rank p. What will be the rank of BA? ($p < m < n$).

1. m	2. p
3. n	4. p + m

14. A square matrix A is said to be idempotent if $A^2 = A$. An idempotent matrix is non-singular if and only if

1. All eigenvalues are real

2. All eigenvalues are nonnegative.

3. All eigenvalues are either 1 or 0

4. All eigenvalues are 1.

15. The polynomial $x^3 + 5x^2 + 5$ is

1. irreducible over ¢ but reducible over ¢ $_5$
2. irreducible over both ¢ and ¢ $_5$
3. reducible over ¢ but irreducible over ¢ $_5$
4. reducible over both ¢ and ¢ $_5$

16. Euler-Lagrange equation, which extremizes the functional

$\iint_D \left\{ \left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right\} dxdy$ with $z = f(x, y)$ on the boundary C of the domain D, where f(x, y) is a given function, constitutes the well known problem, known as

1. Cauchy problem
2. Dirichlet problem
3. Neumann problem
4. Robin or the third boundary value problem

17. Let the minimal polynomial of a linear transformation T from ¡ ^{4} to ¡ ^{4} be $x^2 + x + 1$ Then,

1. all eigenvalues of T are real.
2. T has exactly one real eigenvalue.
3. T has exactly 2 real eigenvalues.
4. T has no real eigenvalue.

18. Let A be a symmetric idempotent matrix. Which of the following is not true?

1. $\det(A) \neq 0 \Rightarrow A = I$

2. Trace A = Rank A

3. If the i^{th} diagonal element of A is zero, at least one element in the i^{th} row is non-zero.

4. Every latent/characteristic root of A is either 0 or 1.

19. Let m be a natural number such that 3^m divides 25! but 3^{m+1} does not divide 25!, Then *m* is equal to

1. 10	2. 8
3. 25	4. 20

20. In ¢ [i]

1. 5 and 6 are irreducible

2. 5 is irreducible but 6 is reducible

3. 5 is reducible but 6 is irreducible

4. neither 5 nor 6 is irreducible

21. Let $S = \{z \in £ \mid |z+i| > |z-i|\}$. Then

1. Im (z) > 0 for z $\in$ S

2. Im (z) < 0 for all Z $\in$ S

3. Re (z) > 0 for all Z $\in$ S

4. Re (z) < 0 for all z $\in$ S

22. Let $a_n = n^n + \frac{(-1)^n}{n}$, n = 1,2,…. Then the sequence (a_n)

1. is convergent but not Cauchy.
2. is Cauchy but not convergent.
3. is not Cauchy but bounded.
4. does not have a convergent subsequence.

23. Let $a_n = (-1)^n \sin\left(\frac{n\pi}{2}\right)$, n = 1, 2,…. Then

1. *lim inf* $a_n = -1$ and lim sup $a_n = 1$
2. *lim inf* $a_n = 0$ and lim sup $a_n = 1$
3. *lim inf* $a_n = -1$ and lim sup $a_n = 0$
4. *lim inf* $a_n = 1$ and lim sup $a_n = -1$

24. If T(z) $= \frac{az+b}{cz+d}$ where $ad - bc \neq 0$, is a Mobius transformation, the ∞

1. is always a fixed point of T.
2. is a fixed point of T and only if c= 0.
3. is never a fixed point.
4. is a fixed point of T if and only if T is identity

25. If $f : £ \to £$ ' is a non constant entire function then the image of f

1. is always equal to £ .

2. is a connected open subset of £ .

3. is a compact subset of £ .

4. $\lim_{2\to\infty} f(z) = \infty$ always.

26. Let $f(z) = \frac{z - \sin z}{z^2}, z \neq 0 and C(t) = e^n, - \leq 2\pi$

Then $\int_C f(z)dz$ is

1. 0
2. 1
3. –1
4. π

27. A pencil manufacturing company packs pencils in boxes, each box containing 10 pencils. If 10% of pencils produced by the company are defective, then the number of defective pencils in a randomly picked box has a

1. symmetric distribution.

2. positively skewed distribution.

3. negatively skewed distribution.

4. bimodal distribution.

28. Let $f(z) = \frac{\tan z - z}{z^3}$ for $z \neq 0$. Then z = 0 is

1. a removable singularity of f.

2. a pole of order 2 of f.

3. a pole of order 1 of f.

4. a pole of order 3 of f.

29. Any function that gives an extremum of the functional

$\iint_D \left\{ \left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 + 2zf(x, y) \right\} dxdy$ must satisfy the so called

1. harmonic equation
2. heat equation
3. wave equation
4. Poisson equation

30. The first order system of equations equivalent to

$x^n + p(t)x' + q(t)x = 0$ is

$\frac{dX}{dt} = A(t)X$ with $X\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, and $\frac{dX}{dt} = \begin{pmatrix} x_1^t \\ x_2^t \end{pmatrix}$ where A (t) is the 2×2 matrix

1. $\begin{pmatrix} 1 & 0 \\ -p & -q \end{pmatrix}$
2. $\begin{pmatrix} 0 & 1 \\ -q & -p \end{pmatrix}$
3. $\begin{pmatrix} 0 & 1 \\ q & p \end{pmatrix}$
4. $\begin{pmatrix} -p & -q \\ 1 & 0 \end{pmatrix}$

31. Consider the boundary value problem (BVP) $y'' + \lambda y = 0$, on [0, 2π], $\wedge \in \mathbb{R}$ with boundary conditions y(0) = y(2π) = 0. Then the above BVP has

1. only trivial solution for every $\lambda \in \mathbb{R}$.
2. has a unique solution for all $\lambda \in \mathbb{R}$.
3. has nontrivial solution for a countable set of values of $\lambda \in \mathbb{R}$.
4. has a nontrivial solution for every $\lambda \in \mathbb{R}$.

32. The function f(x, y) = x |y|, (x, y) $\in \mathbb{R} \times \mathbb{R}$

1. satisfies a Lipschitz condition on the strip $-1 \le x \le 1, -\infty < y < \infty$.
2. satisfies a Lipschitz condition on the strip

   $-\infty < x < \infty, -1 \le y \le 1$.
3. does not satisfy a Lipschitz condition on $-1 \le x \le 1, -1 \le y \le 1$.
4. does not satisfy a Lipschitz condition at any point on $\mathbb{R} \times \mathbb{R}$.

33. Consider the linear differential equation

$y^{(3)} + 3xy^{(2)} + 4y^{(1)} + 2x^2 y = \sin x, \left(x \in [0,1]\right)$ Then the set of solutions of the above question.

1. is a linear space of infinite dimension.
2. is a linear space of dimension 3.
3. is not a linear space.
4. is a linear space of dimension less than 3.

34. The partial differential equation $(u_x)^4 u_{xx} + u_{yy} + (u_y)^2 = 0$ is

1. linear and of order 4
2. quasilinear and of order 2
3. quasilinear and of order 4
4. linear and of order 2.

35. Consider the linear programming problem:

Maximize $2x_1 - 3x_2 + x_3 + 4x_4$

subject to

$x_1 - x_2 + x_4 \geq 4$

$2x_1 - x_3 + 2x_4 \leq 1$

$x_1 + x_2 + x_2 \leq 2$

$x_1 \geq 0, x_2 \geq 0, x_3 \leq 0, x_4 \geq 0.$

If (y_1, y_2, y_3) is a dual feasible solution, then

1. $y_1 > 0$ and $y_2 > 0$.
2. $y_1 \leq 0$ and $y_2 \geq 0$.
3. $y_2 < 0$ and $y_3 < 0$.
4. $y_2 < 0$ and $y_3 \geq 0$

36. The second order partial differential equation

$u_{xx} + 4u_{xy} + (\cos x)u_{yy} + e^x u_x + e^y u_y = 0$ has

1. exactly one characteristic curve passing through every point
2. exactly one characteristic curve passing through the point (0, 0).
3. Two distinct characteristic curves passing through every point.
4. Three distinct characteristic curves passing through the point (1, 3).

37. The number of distinct homoporphisms from $\mathbb{Z}_{12}\ to\ \mathbb{Z}_{25}$

1. one
2. two
3. three
4. four

38. The number of 5-Sylow subgroups of S_6 is

1. 16
2. 6
3. 36
4. 1

39. Let G be a group of order 14 such that G is not abelian. Then the number of elements of order 2 in G is equal to

1. 7
2. 6
3. 13
4. 2

40. The solution of the Caughy problem $u_x + u_y = 0, u(x,0) = e^x$ is

1. $u(x,y) = e^{x+y}$
2. $u(x,y) = e^{x+e^{2y}}$
3. $u(x, y) = e^{x-y}$

4. $u(x,y)=\frac{1}{2}\left[e^{x+y}+e^{x-y}\right]$

41. The equation

$$u(x)=f(x)+\int_{a}^{a}K(x,t)u(t)dt$$ is

1. a Volterra's linear integral equation of the first kind.
2. A Fredholm's linear integral equation of the first kind.
3. a Volterra's linear integral equation of the second kind.
4. a Fredholm's linear integral equation of the second kind.

42. Euler's equation of motion for a rigid body about a fixed point, in the absence of external forces, and $I_{xx} = 1_{yy}$ imply that the z-component of the angular velocity is

1. a function of time
2. a constant other than zero and unity
3. zero
4. unity.

43. Suppose that the minimal polynomial of a linear map $T:¡^5 \to ¡^5$ is $x^2(x^3-1)$.

Then,

1. $T \equiv 0$
2. Det (T) = 0
3. Det(T) $\neq$ 0
4. T is onto

NOTE :- Question after this are for statistics students only and therefore not included here