Part-B

1. The ODE for the family of straight lines $y = \frac{x}{t} + t$ where t is a parameter that takes positive values is

   1. $yy' = xy'^2 + 1$ and has no singular solutions.
   2. $yy'^2 = xy' + 1$ and has no singular solutions.
   3. $yy' = xy'^2 + 1$ and has exactly one singular solution.
   4. $yy'2 = xy' + 1$ and has exactly one singular solution

2. The solution of the initial value problem

   $$y'' + y = \sec^3 \times \text{ on } [0, \pi/2) \text{ with } y(0) = 1, y'(0) = 0 \text{ is}$$

   1. $y = \int_0 \sin(t - x)\sec^3 t\, dt + \cos x$
   2. $y = \int_0 \sin(t - x)\sec^3 t\, dt + \sin x$
   3. $y = \int_0 \cos(t - x)\sec^3 t\, dt + \sin x$
   4. $y = \int_0 \cos(t - x)\sec^3 t\, dt + \cos x$

3. On a certain domain D $\subset$ ¡ ^{2} it is given that u is harmonic and u = 1 on the circle {(x,y) | $x^2 + y^2 = 1$} $\subseteq$ D and u = 2 on the circle {(x, y) | $x^2 + y^2 = 9$ } $\subseteq$ D

Then which of the following hold:

   1. D can be whole of R2

2. On the largest possible domain D the function u is bounded.

3. u takes value 3 on the $x^2 + y^2 = 4$

4. u takes value 3 on $x^2 + y^2 = 81$

4. Given 4 points A, B, C and D (in that order) along a parallelogram whose sides have slopes $\pm \frac{1}{c}$ in the (x, t) plane and u is a solution of the wave equation

$u_u - c^2 u_{xx} = 0$.

Suppose u(A) = $\frac{1}{2}$, u(C) = $\frac{1}{4}$, u(B) = $\frac{2}{3}$. The u (D) equals.

1. $\frac{1}{6}$
2. $\frac{1}{12}$
3. $\frac{1}{3}$
4. $\frac{7}{12}$

5. The solution of the PDE $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$ with initial data $u(x,1) = x(1-x)$ is

1. $xy^2 - x^2y$
2. $xy^3 - x^3y$
3. $xy^2 - x^2y^2$
4. $xy - x^2y^2$

6. Let $P_n(x)$ be the Taylor polynomial of degree $n \geq 0$ for the function e^x about x = 0. Then the error in this approximation is

1. $\frac{x^n}{n!}e^{\xi}$ for some $0 < \xi < x$

2. $\dfrac{x^n}{(n+1)!}e^{\xi}$ for some $0 < \xi < x$

3. $\dfrac{x^{n+1}}{n!}e^{\xi}$ for some $0 < \xi < x$

4. $\dfrac{x^{n+1}}{(n+1)!}e^{\xi}$ for some $0 < \xi < x$

7. Let $\{e_1, e_2, e_3\}$be a basis of a vector space V over ¡ . Consider the following gets:

A = $\{e_2, e_1 + e_2, \quad e_1, e_2 + e_3\}$

B = $\{e_1, e_1 + e_2, \quad e_1, e_2 + e_3\}$

C = $\{e_2, e_1 + e_2, \quad e_1, e_2 + e_3\}$

1. A and B are bases of V.
2. A and C are bases of V.
3. B and C are bases of V.
4. Only B is a basis of V.

8. Let A be an n × m matrix and b = $(b_1, b_2, b_n)^1$.

Consider the following statements:

(a) If rank A = n, the system has a unique solution

(b) If rank A < n, the system has infinitely many solutions.

(c) If b = 0, the system has at least one solution.

Which of the following is correct?

1. (a) and (b) are true.

2. (a) and (c) are true.

3. only (a) is true.

4. only (b) is true.

9. Let $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ Then the eigenvalues of A are

1. 2, a and 0

2. 2, a+1 and 1–1

3. 2, –1 and –1

4. 1, –1 and 0

10. Let $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ be such that A has real eigenvalues. Then

1. $\theta = n\pi$ for some integer n

2. $\theta = 2n\pi + n\pi/2$ for some integer n

3. There is no restriction on θ.

4. $\theta = 2n\pi+\pi/4$ for some integer n.

11. Let A = (an)be an n×n matrix with real entries such that the sum of all the entries in each row is zero. Consider the following statements:

(a) A is non-singular.

(b) A is singular.

(c) 0 is an eigenvalue of A.

Which of the following is correct?

1. Only (a) is true.
2. (a) and (c) are true.
3. (b) and (c) are true.
4. Only (c) is true.

12. Let $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ Then the minimal polynomial of A is

1. $\lambda^2 - 4\lambda - 5$
2. $\lambda^2 + 5\lambda + 4$
3. $\lambda^3 - 3\lambda^2 - 9\lambda - 5$
4. $\lambda^3 + 3\lambda^2 - 9\lambda + 5$

13. Let $A = (a_{ij})$ be an n×n matrix such that $a_{ij} = 3$ for all i and j. Then the nullity of A is

1. n – 1
2. n – 3
3. n
4. 0

14. Let A be a non-zero matrix of order 8 with $A^2 = 0$, Then one of the possible value for rank of A is

1. 5   2. 6

3. 4   4. 8

15. Let W = $\{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ with standard dot product in $\mathbb{R}^3$. Then an orthonormal basis of W is.

1. $\{(1,o,o)(0,1,o),(0,0,1)\}$
2. $\left\{\frac{1}{\sqrt{3}}(1,1,1), \frac{1}{\sqrt{2}}(1,0,-1), \frac{1}{\sqrt{2}}(0,1,-1)\right\}$
3. $\left\{\frac{1}{\sqrt{2}}(1,0,-1), \frac{1}{\sqrt{2}}(1,0,-1), \frac{1}{\sqrt{2}}(0,1,-1)\right\}$
4. $\left\{\frac{1}{\sqrt{6}}(1,1,-2), \frac{1}{\sqrt{2}}(1,-1,0)\right\}$

16. Let $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}$. Then the equation $X^tAX = 1$, where $X = (x_1, x_2, x_3)^t$ represents

1. a hyperboloid of two sheets.
2. an ellipsoid.
3. a pair of planes.
4. a paraboloid.

17. Let w denote a complex cube root of unity. The cube roots of the complex number -2 + 2i are

1. $1-i, (1-i)w, (1-i)w^2$
2. $1+i, (1+i)w, (1+i)w^2$

3. $(1-i)^{-1}, (1-i)^{-1} w, (1-i)^{-1} w^2$

4. $(1+i)^{-1,}, (1+i)^{-1} w, (1+i)^{-1} w^2$

18. Let $f(z) = \sum_{n=0}^{\infty} n^2 (1-\cos z)^n$ have the Taylor series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$ around 0. Then

1. $a_2 = 0, a_3 = -1$

2. $a_2 = -\frac{1}{2}, a_3 = -1$

3. $a_2 = \frac{1}{2}, a_3 = 0$

4. $a_2 = 0, a_3 = 0$

19. Let f(x, y) = u(x, y) + i v(x, y), where u (x, y) = $x^2 - y^2$ and v(x, y) = - 2xy. Then

1. f is complex differentiable

2. u, v are differentiable and $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

3. u, v are differentiable and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

4. The function g(z) = f(z) is complex differentiable.

20. Let $f : £ \to £$ be a non constant holomorphic function. Which of the following can occur?

1. Range of $f \subseteq \{ w \in £ : |w| < 1 \}$

2. Range of $f \subseteq \{w \in £ : Rew > 0\}$

3. Range of $f \subseteq \{w \in £ : 1 < |w| < 2\}$

4. Range of $f \subseteq \{w \in £ : w \neq 0\}$

21. Let f be an analytic function on the open unit disc D taking values in D such that f(0) = 0. Which of the following is possible?

1. f(1/2) = 3/4

2. f(1/2) = 1/3

3. f(1/2) = (3/4) 1

4. f(1/2) = e^{i}

22. Let γ be the curve in the complex plane

$$\gamma(t) = \begin{cases} -1 + e^{2\pi it} & -1 \leq t \leq 0 \\ e^{2\pi it} & 0 \leq t \leq 1 \end{cases}$$

Then $\frac{1}{2pi}\int_{r} \frac{1}{z-a} dz$ equals

1. 0 for all $a \in £$

2. 0 if and only if

$a \notin \{z : |z+1| < 1\} \text{ U } \{z : |z| < 1\}$

3. 0 for all $a \notin \{z : |z+1| < 1\} \text{ I } \{z : |z| < 1\}$

4. 1 if and only if

$a \notin \{z : |z+1| < 1\} \text{ U } \{z : |z| < 1\}$

23. Let $w = \dfrac{az+b}{cz+d}$ map the points $-2, 2$

and ∞ to i, 1, -i. Let D $= \{z : |z| < 1\}$

The points i, 1+i will be mapped:

1. inside **D**
2. outside **D**
3. one inside **D** and the other outside **D**
4. both on the boundary of **D**

24. 6 boys and 6 girls leave their umbrellas outside the classroom. Then the number of ways in which every person picks up one umbrella in such a way that all the boys get girls umbrellas are

1. 6   2. 12!
3. 12   4. $(6!)^2$

25. 5 people take a lift at the ground floor of a building with three floors. The number of ways in which at least one person gets out at each of the three floors 1, 2 and 2 is

1. 150   2. 21
3. 243   4. 10

26. Let G be the group of symmetries of a rhombus. Then G is isomorphic to

1. the trivial group {e}.   2. ¢$_2$
3. ¢$_2 \times$ ¢$_2$ 4. ¢$_4$

27. Let $\alpha = (1,3,5,7,9,11)$ and $\beta = (2, 4, 6, 8)$ be two permutations in S_{100}, where S_{100} denotes the symmetric group on $\{1,2, \ldots, 100\}$. Then the order of $\alpha\beta$ is

1. 4 2. 6
3. 12 4. 100

28. The number of groups of order 121, up to isomorphism, is

1. 1 2. 2
3. 11 4. 10

29. Let G a non-abelian group of order 21. Let H be a Sylow 3-subgroup and K be a Sylow 7-sub group of G. Then

1. H and K are both normal in G
2. H is normal but K is NOT normal in G
3. K is normal but H is NOT normal in G.
4. Neither H nor K is normal in G.

30. Let $f(x) \in \mathbb{Z}_5[x]$ be a polynomial such that $\mathbb{Z}_5[x]/\langle f(x)\rangle$ is a field, where $\langle f(x)\rangle$ denotes the ideal generated by $f(x)$. Then one of the choices for $f(x)$ is

1. x^2+1 2. x^2+3
3. x^3+1 4. x^3+3

31. Let A = {1,2,3…,2n} where $n \geq 2$. Let B, a subset of A, be of size m. Then the smallest value of m for which the set B must have two co-prime numbers is.

1. 2
2. n – 1
3. n
4. n + 1

32. The ODE $y''+y=\sin^3 x$ is solved by the method of undetermined coefficients. The simplest form of the particular solution (i.e. one with least number of undetermined coefficients) is

1. $\mathrm{A}\sin x \div \mathrm{B}\cos x+\mathrm{C}\sin 3x+\mathrm{D}\cos 3x$
2. $\mathrm{A}\sin x+\mathrm{B}\cos x+\mathrm{C}x\sin 3x+\mathrm{D}x\cos 3x$
3. $\mathrm{A}x\sin x+\mathrm{B}x\ \cos x+\mathrm{C}\sin 3x+\mathrm{D}\cos 3x$
4. $\mathrm{A}\sin x+\mathrm{B}\ \cos x+\mathrm{C}x\sin x+\mathrm{D}x\cos+\mathrm{E}\sin 3x+\mathrm{F}\cos 3x.$

33. Let $y_1(x)$, $y_2(x)$ be two eigenfunctions corresponding to distinct eigenvalues of the Sturm-Liouville problem

$$y''+\lambda\rho(x)y=0;$$

with boundary condition y(0) = 0 = y(1), where q(x) is positive and $y_1(x) > 0$ on (0,1). Which of the following is NOT true?

1. $\int_0^1 q(x)y_1(x)y_2(x)dx=0$
2. $y_1(x)$ and $y_2(x)$ are linearly independent.
3. $y_2(x)$ must vanish at most once on (0, 1).
4. $y_2(x)$ must vanish at least once on (0, 1).

34. For an approximate computation of $\frac{1}{\alpha}$, based on Newton-Raphson method, where α is a positive real number, the function f(x) in the interaction formula $x_{n+1} = f(x_n), n \geq 0$ has the form

1. $x(1-\alpha x)$
2. $x(2-\alpha x)$
3. $x(1-\alpha)$
4. $x(2+\alpha x)$

35. Let $J[y] = \int_{-1}^{1} (y'e^y + xy^2)dx$ be a functional defined on $C^1[-1,1]$

Then the variation of J[y] is

1. $\int_{-1}^{1} (y''e^y + e^y y' + 2_0 xyy')dx$
2. $\int_{-1}^{1} (y'^2 e^y + y^2 + 2xyy')dx$
3. $\int_{-1}^{1} [(y'e^y + 2xy)\delta y + e^y \delta y']dx$
4. $\int_{-1}^{1} [(y'e^y + 2xy + y^2)\delta y + e^y \delta y']dx$

36. Consider the functional $J[y] = \int_{1}^{2} (xy'^4 - 2yy'^3)dx$ defined on

$S = \{y / y \in C^1[1,2]$ and $y(1) = 0, y(2) = 1\}$

1. a weak minimum on $y = x - 1$
2. a strong minimum on $y = x - 1$
3. a weak maximum on $y = x - 1$

4. a strong maximum on $y = x–1$

37. limsup $\left\{ \frac{(-1)^n}{2^n} : n1, 2, ... \right\}$ is

1. 0
2. $\frac{1}{4}$
3. 1
4. –1

38. Let $P(x) = x^n + 5x^2 + 7x, nodd, n \geq 3$. Fix

$\alpha \in$ ¡ $S = \{x \in$ ¡ $| P(x) = \alpha\}$. Then

1. S is empty
2. S is finite and non empty
3. S is countably infinite
4. S is uncountable

39. Let (x_n) and (y_n) be sequences of real numbers where $x_n = (1)^n$ (sin n) and $y_n = (–1)^n n$. Then

1. (x_n) and (y_n) both have convergent subsequences.
2. None of the two sequences (x_n), (y_n) have a convergent subsequence.
3. (x_n) does not have a convergent subsequence while j(y_n) has.
4. (x_n) has a convergent subsequence while (y_n) does not have.

40. Let $f :$ ¥ $\to$ ¡ be any function such that f takes values in ¢. Assume that lim f(n) exists. Then

1. f is a constant

2. There exists l, m $\in$ ¢ , $l \neq m$ such $f^{-1}(\{m\})$ and $f^{-1}(\{l\})$ are infinite.

3. $f^{-1}($¥$)$ and $f^{-1}(\{\ldots -n, \ldots -2, -1\})$ are infinite.

4. There exists $k \in$ ¥ such that $\lim\limits_{n\to\infty} f(n) = f(k)$

41. Let x be in ¡ . Then the series $\sum\limits_{1}^{\infty} x^n e^{-n}$ is

1. always convergent
2. converges only at x = 0
3. converges if x < e
4. converges if |x|<e

42. Let f : ¡ $\to$ ¡ be given by $f(x) = [x]$, the greatest integer less than or equal to x. Then

1. The points at which f is not continuous is countable.
2. The points at which f is not continuous is ¡ .
3. f is of bounded variation.
4. f is strictly increasing.

43. Let f: $[0,1] \to [0,1]$ be a strictly increasing onto function. Then

1. f is continuous but f is not.
2. f and f^{1} are both continuous.
3. f^{1} is continuous but f is not.

4. f^{1} is not Riemann integral on [0, 1].

44. Let $P(x,y)=a+bx+cy+dx^2+exy+fy^2$ be a polynomial function of two variables such that P(0, 0) is a local minimum. Then

1. $bc \neq 0$
2. $b = 0$ and $c \neq 0$
3. $b \neq 0$ and $c = 0$
4. $b = 0$ and $c = 0$

45. Let $f:\mathbb{R}^2 \to \mathbb{R}$ be such that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at all points. Then

1. The total derivative of f exists at all points of $\mathbb{R}^2$.
2. f is continuous on $\mathbb{R}^2$.
3. The function $f(x, y)$ as a function of x for every fixed y and f(x, y) as a function of y for every fixed x are continuous.
4. All directional derivatives of f exist at all points of $\mathbb{R}^2$.

46. Let C be the standard Cantor's middle third set. Then

1. C is not measurable
2. C is countable and of measure zero
3. C is uncountable and of measure zero.
4. C is uncountable and of positive measure.

47. Let $S = \{(x, y) \in \mathbb{R}^2 : x^2 - y = 0\}$.

Then

1. S is connected but not compact.

2. S is not connected and not compact.

3. S is not connected but compact.

4. S is connected and compact.

48. Let V be the vector space of all 5 × 5 real skew-symmetric matrices. Then the dimension of is

1. 20 2. 15

3. 10 4. 5

49. Which of the following is a degenerate kernel?

1. $k(x,t)=\sum_{n=1}^{\infty} x^n t^n$ 2. $k(x,t)=e^{|x|+1}$

3. $k(x,t)=e^{|x-t|}$ 4. $k(x,t)=e^{xt}$

50. A solution of the integral equation $\int_0^x 3^{x-1}\Phi(t)=3e^x$ is

1. $\Phi(x)=3e^x$

2. $\Phi(x)=1-3e^z$

3. $\Phi(x)=1-x\log 3$

4. $\Phi(x)=x\log 3$

51. In a conservative field of motion, which of the following statement is not correct.

1. generalized coordinates do not depend on time explicitly

2. all forces are derivable from the potential

3. potential energy is explicitly a function of time.

4. the total energy is constant

52. If the Lagrangian of a dynamical system is $L = \frac{1}{2} ml^2\theta^2 - mgl\cos\theta$ then the corresponding Hamiltonian is

1. $H = \frac{1}{2} ml^2\theta^2 + mgl\cos\theta$

2. $H = \frac{1}{2} ml^2\theta^2 - mgl\cos\theta$

3. $H = \frac{-1}{2} ml^2\theta^2 - mgl\cos\theta$

4. $H = \frac{-1}{2} ml^2\theta^2 + mgl\cos\theta$

NOTE:- Questions after this are for statistics students only and therefore aren't included here.