Part-B

June – 2010

1. Let f, $g:£ \to £$ be given by f(z) = |z|^2; g(z)=z^2. Then

   1. f and g are analytic on £
   2. f is not analytic and g is analytic on £ .
   3. f is analytic and g is not analytic on £ .
   4. neither f nor g in analytic on £ .

2. Let $f(x)=\lim_{n\to 0}$

   $\tan\left(\frac{nx+x^n}{n}\right), x\in\left[0,\frac{1}{2}\right]$.. Then the function f is

   1. Continuous but not uniformly continuous.
   2. Uniformly continuous.
   3. not continuous.
   4. not defined.

3. Let $f:[0,1]\to ¡$ be the function given by

   $f(x)=1$ if $0\le x\le 0.5$

   2 if 0.5 < x $\le$ 0.7

   3 if 0.7 < x $\le$ 1

   Then

   1. f is not Riemann integrable and f is not Lebesgue integrable

2. f is Riemann integrable and $\int_0^1 f(x)\,dx = 1.8^0$

3. F is Riemann integrable and $\int_0^1 f(x)\,dx = 2.1$

4. f is not Riemann integrable but f is Lebesgue integrable

4. Let V be the vector space of polynomials of degree < 5 over ¡ . Let D : V $\rightarrow$ V be the derivative map P $\rightarrow$ p^1. Then

   1. 0 is the only eigenvalue of D

   2. 1 is an eigenvalue of D

   3. 5 is an eigenvalue of D

   4. D is invertible

5. Let A = Q, the set of rational numbers, and B = [e, π]. Then

   1. A is connected and B is not connected

   2. A is not connected and B is not connected

   3. A is not connected and B is connected

   4. A is connected and B is connected

6. The form $f(x,y) = x^2 - 4xy + 5y^2$ on ¡ ^{2} is

   1. symmetric and positive definite

   2. not symmetric, but positive definite

   3. symmetric, but not positive definite

4. neither symmetric positive definite

7. Given a square-matrix A over ¡ with characteristic polynomial $(x-1)^2 x$ and number of distinct Jorden canonical forms is

1. 1
2. 2
3. 3
4. 4

8. If $f_n(x) = e^{\frac{-x^3}{n}}, x \in [0,1], n = 1,2,...,$ the $\lim_{n\to\infty} \int_0^1 f_n(x)\,dx$

1. does not exist
2. exists and equals 1.
3. exists and equals 0
4. exists and equals e.

9. Let $A: £^n \to £^n$ be a Hermitian linear map, $Av_1 = v_1$ and $Av_2 = 2v_2 \neq 0$, $v_2 \neq 0$. Then

1. $\|v_1 + v_2\|^2 = \|v_1\|^2 + \|v_2\|^2$
2. $\|v_1 + v_2\| < \|v_1\| + \|v_2\|$
3. $\|v_1 - v_2\| = \|v_1\| - \|v_2\|$
4. $\|v_1 - v_2\| = \|v_1\| + \|v_2\|$

10. A Class of 10 students has to first select a committee of 3 students and among the 3 students selected, one student is declared the president, another student is declared the vice-president and the

third person is selected as secretary of the committee. In how many way can this be done?

1. $\frac{10!}{3!}$ ways

2. 120 ways

3. 720 ways

4. 10! ways

11. Let $f(x)=\frac{\sin x}{x}$ and $g(x)=\frac{\sin x}{\sqrt{x}}, x\in$ [0, 1]. Then the improper integrals

1. $\int_0^1 g(x)dx$ exists and $\int_0^1 f(x)dx$ does not exist

2. $\int_0^1 f(x)dx$ exists and $\int_0^1 g(x)dx$ does not exist

3. $\int_0^1 f(x)dx$ and $\int_0^1 g(x)dx$ exist

4. $\int_0^1 f(x)dx$ and $\int_0^1 g(x)dx$ do not exist

12. Let E = $\{2n+r \mid r\in \mathbb{Q} \cap [0,1/2]$ and n $\in \mathbb{N}$ }. Then the boundary of E in $\mathbb{R}$ is

1. $\bigcup_{n=1}^{\infty}[2n,2n+1/2]$

2. E

3. $\mathbb{R}$

4. even integers

13. Let $V = \left\{ \left(x_1, ..., x_{100} \right) \in \mathbb{R}^{100} : x_1 = ... = x_{50} \text{ and } x_{51} + x_{52} + ... + x_{100} = 0 \right\}$

Then

1. dim V = 98

2. dim V = 59

3. dim V = 49

4. dim V = 50

14. The system of simultaneous linear equations

$x + y + z = 0$

$x - y - z = 0$

has

1. no solution in $\mathbb{R}^3$.

2. a unique solution in $\mathbb{R}^3$.

3. infinitely many solutions in $\mathbb{R}^3$.

4. More than 2 but finitely many solutions in $\mathbb{R}^3$.

15. Let $P(3) = \left\{ a_0 + a_1 x + a_2 x^2 + a_3 x^3 \mid a \in \mathbb{R}, i = 0,1,2,3 \right\}$

Under the standard operation of addition (+) and scalar multiplication (.), P(3) is

1. not a vector space

2. a vector space of infinite

3. dimension

4. a| vector space of dimension 4

16. If A is a real 2×2 matrix such that $A^2 - A = 0$, then

1. either A = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or A = $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

2. there are infinitely many such matrices A.

3. there are only finitely many such matrices A.

4. A has to be a diagonal matrix

17. Let A and B be upper triangular matrices given by

$$A = \begin{pmatrix} 1 & & \cdot \\ & 2 & \\ 0 & \text{O} & n \end{pmatrix} and\ B = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ \cdot & \text{O} & n-1 \end{pmatrix}$$

Then

1. A is invertible and B is singular

2. A is singular and B is invertible

3. both A and B are invertible

4. Neither A nor B is invertible

18. Given that a 3 × 3 matrix satisfies the equation $A^3 - A^2 + A - I = 0,$ the value of A^4 is

1. not computable from the given data

2. $-A^3 - A^2 + A - I = 0$

3. 0

4. I

19. Let A and B be finite sets of m and n elements respectively. Then the number of functions f: A$\rightarrow$B is

1. mn
2. m+n
3. m^n
4. n^m

20. Let A be the n × n matrix with all entries equal to 1. The eigenvalues of A are

1. 0 with multiplicity (n-1) and n with multiplicity 1

2. 0 with multiplicity 1 and n with multiplicity (n-1)

3. 0 with multiplicity 1 and 1 with multiplicity 1

4. with multiplicity 1 and 1 with multiplicity 1.

21. Let $a_n = \frac{4^{3n}}{3^{4n}} n = 1, 2, \ldots$ Then the sequence (a_n)

1. is unbounded

2. is bounded but not convergent

3. converges to 0

4. converges to 1

22. Sup $\left\{\left(\sqrt{n+1}-\sqrt{n}\right)\right\}$ and

1. $\sqrt{2}+1$ and 0 respectively

2. $\frac{1}{\sqrt{2}+1}$ and 0 respectively

3. both equal to 0

4. both equal to $\frac{1}{\sqrt{2}+1}$

23. Let (x_n) be a Cauchy sequence of real members. Then the sequence $(\cos x_n)$ is

1. unbounded

2. bounded but not Cauchy

3. Cauchy but not bounded

4. Cauchy

24. The number of generators of a cyclic group of order 12 is

1. one
2. two
3. three
4. four

25. The set $\left\{x \in \left(-\pi, \pi\right) : \sin x \,|> 1/2\right\}$ is

1. an open interval

2. a union of finitely many disjoint open intervals

3. a union of countably infinitely many disjoint open intervals

4. a union of uncountably many disjoint open intervals

26. Up to an isomorphism, the number of groups of order 33 is

1. 3
2. 11
3. 1
4. Infinitely many

26. Let $R =$ ¤ $[x]$. Let I be the principal ideal $\langle x^2+1\rangle$ and J be the principal ideal $\langle x^2\rangle$. Then

1. R/I is a field and R/J is a field
2. R/I is an integral domain and R/J is a field
3. R/I is a field and R/J is a PID
4. R/I is a field and R/J is not an integral domain

27. The polynomial ring ¢ [x] is

1. a Euclidean domain but not a PID
2. a PID but not Euclidean
3. Neither PID nor Euclidean
4. both PID and Euclidean

28. The polynomial $x^3-7x^2+15x-9$ is

1. irreducible over both ¢ and ¢ $_3$
2. irreducible over ¢ but reducible over ¢ $_3$
3. reducible over ¢ but irreducible over ¢ $_{3.}$
4. reducible over both Z and ¢ $_{3.}$

29. A permutation a of {1, 2,..., n} is called a derangement if $\alpha(i) \neq i$ for every i. Let d_n denote the number of derangements of {1, 2, ...n}. Then d_4 is equal to

1. 3
2. 9
3. 12
4. 24

30. The equation $z^3 + 2z + 50 = 0$ has

1. a solution in $\{z \in \mathbb{C} : |z| < 1\}$
2. no solution in $\{z \in \mathbb{C} : |z| < 1\}$
3. infinitely many solutions in $\mathbb{C}$.
4. a solution in $\{z \in \mathbb{C} : 1 < |z| < 2\}$

31. The number of subfields of a finite field of order 3^{10} is equal to

1. 4
2. 5
3. 3
4. 10

32. The Mobius transformation mapping (0, 1, ∞) into (1, ∞, 0) respectively, maps

1. real numbers into real numbers
2. purely imaginary numbers into purely imaginary numbers
3. unit circle into unit circle
4. real numbers into unit circle

33. $f : \mathbb{C} \to \mathbb{C}$ is analytic and f(z) is real for all z in C. Then

1. f is bounded

2. f is not bounded

3. such an f does not exist

4. f has a singularity at z = ∞

34. If γ is the circle of radius $\frac{1}{2}$ with center 1, then $\int_{.0} \frac{dz}{z^2 - z}$ is

1. 0
2. 1
3. 2πi
4. $\frac{1}{2\pi i}$

35. If γ is the square with vertices 0, 1, 1 + i and i, then $\int_z dz$ is

1. 1
2. 0
3. 4
4. 2πi

36. Consider the linear homogeneous differential equation

$y^{(n)} + P_1(x) y^{(n+1)} + ... + P_n(x) y = 0$ on [0, 1], where P_1,...P_n are continuous real valued functions on [0, 1]. Then the set of solutions of the above equation.

1. is a linear space of infinite dimension

2. is a linear space of dimension n

3. is a linear space of dimension less than n

4. is not a linear space

37. The singularities of $\frac{1}{z-z^3}$ in the extended complex plane are

1. 3 poles of order 1
2. 1 pole of order 1 and 1 pole of order 2
3. 3 poles of order 1 and an essential singularity at ∞
4. 3 poles of order 1 and a removable singularity at ∞

38. The eternizing curve for the functional

$F(y,z)dx$ with $F_{y'y'}Fz'z' - (fy'z')^2 \neq 0,$

1. is uniquely determined
2. is a member of an infinite family of curves lying in a plane
3. belongs to a family of curves in space
4. does not exist

39. Let u and v be two solutions of $y^{(2)} + P(x)y^{(1)} + Q(x)y = 0$ on [a, b]. Let W(u, v) denote the Wronskian determinant of u and v. Then

1. W (u, v) vanishes at a point $x_0 \in [a,b] \Rightarrow$ u and v are linearly dependent.
2. W(u, v) vanishes identically on $[a,b] \Rightarrow$ u and v are linearly independent.
3. W(u, v) vanishes at a point on [a, b] but does not vanish identically.

4. W(u, v) does not vanish at any point on [a, b] but u and v are linearly dependent.

40. The Legendre's equation

$$\left[\left(1-x^2\right)y^{(1)}\right]^{(1)} p\left(p+1\right)y=0$$

on the interval [0, 1] has

1. both 0 and 1 as regular singular points
2. both 0 and 1 as regular points
3. a regular singular point at 0 and a regular pint at 1
4. a regular point at 0 and a regular singular point at1.

41. An extremum of the functional

$$\int_0^1 \left[\left(y'\right)^2 + 12xy\right]dx, \quad y\left(0\right)=0, y\left(1\right)$$

= 1 can occur only along the curve

1. $x\left(1-x\right)e^x$
2. $\left(1-x\right)x^2$
3. x^2
4. x^3

42. For arbitrary real valued smooth functions f and g, the function u defined as

$u\left(x,t\right)= f\left(x+t\right)+g\left(x-t\right)$ is a general solution of

1. $u_u + u_x = 0$

2. $u_u + u_{xx} = 0$

3. $u_u + u_{xx} = 0$

4. $u_u - u_{xx} = 0$

43. Let $u(x,t)$ solve the heat equation $u_t - u_{xx} = 0, 0 < x < \pi, 0 < t < \infty$

$u(x,0) = \sin x, 0 < x < \pi \;\; u(0,t) = u(\pi \; t) = 0 \;\; t > 0$ The u

1. is unbounded in $(0,\pi)\times(0,\infty)$

2. takes both positive and negative values in $(0,\pi)\times(0,\infty)$

3. is negative in $(0,\pi)\times(0,\infty)$

4. $u(x,t) \le e^{-t}$, for all $(x,t) \in (0,\pi)\times(0,\infty)$

44. Given the following data

i	0	1	2
x_i	0	1	2
$f(x_i)$	1	3	2

an approximate value of f (0.5), using Newton's interpolation, is

1. 1.234
2. 1.832
3. 2.301
4. 2.375

45. Let $f \in C^4\left[x_{-1,} x_1\right], f_i = f\left(x_i\right) and f_i' = f$

$'(x_i)$, and $f_i'' = f''(x_i)$ and so forth, where $x_i - x_0 + ih, i = 0, \pm 1$ with $h > 0.$

Then there exists a point $\xi \in \left(x_{-1}, x_1\right)$

such that

$f_0'' = \frac{1}{h^2}\left(f_{-1} - 2f_0 + f_1\right) + e(\xi),$ where the error e (ξ) is given by

1. $hf''(\xi)/2$

2. $-h^2 f'''(\xi)/6$

3. $h^3 f^{(IV)}(\xi)/12$

4. $-h^2 f^{(IV)}(\xi)/12$

NOTE :- Questions after this are for statistics students only and therefore aren't included here.